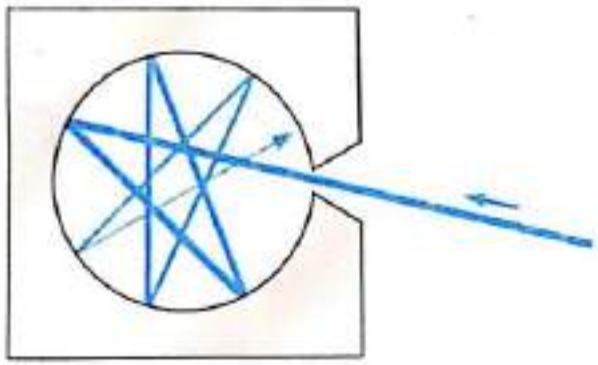
BLACK BODY RADIATION

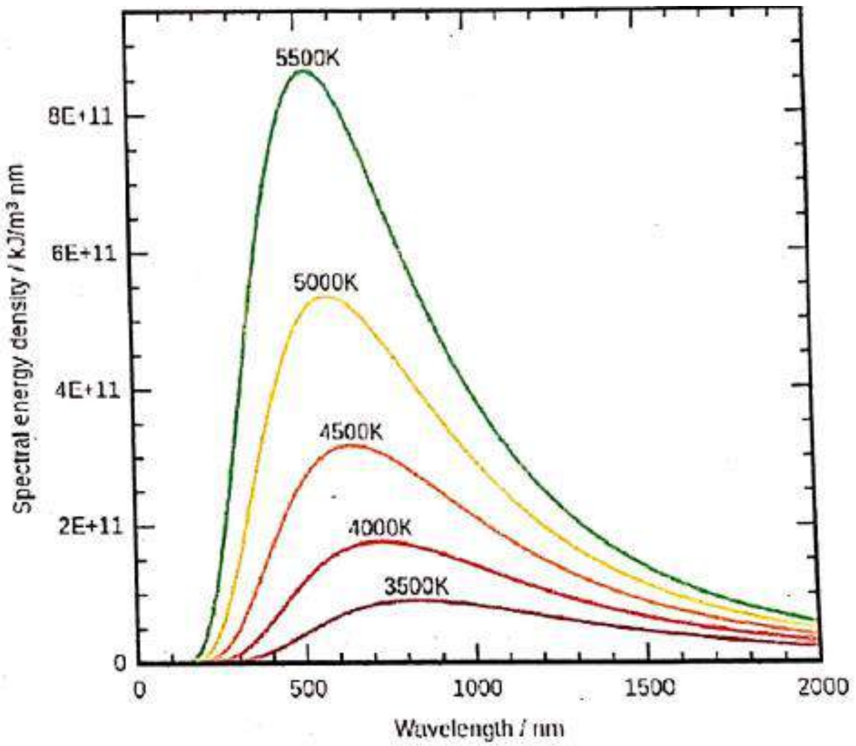
A body at temp. above absolute zero emits radiation in all directions over a wide range of wavelength.



blackbody

A blackbody is a surface that

- completely absorbs all incident radiation
- emits radiation at the maximum possible monochromatic intensity in all directions and at all wavelengths.



Blackbody Spectrum

The emitted radiation is a continuous function of *wavelength.* At any specified temperature, it increases with wavelength, reaches a peak, and then decreases with increasing wavelength.

At any wavelength, the amount of emitted radiation *increases* with increasing temperature.

As temperature increases, the curves shift to the left to the shorter-wavelength region. Consequently, a larger fraction of the radiation is emitted at *shorter wavelengths* at higher temperatures.

DEFINITIONS

Total energy density (u) at any point denotes the total radiant energy for all wavelengths from 0 to ∞ per unit volume around that point. Its unit is Jm^{-3}.

Spectral energy density (u_λ) for the wavelength λ is a measure of the energy per unit volume per unit wavelength. Therefore, $u_\lambda d\lambda$ denotes the energy per unit volume in the wavelength range between λ and $\lambda + d\lambda$. It is related to total energy density through the relation

$$u = \int_0^\infty u_\lambda \, d\lambda$$

Total emissive power E of the surface of the body at a given temperature is defined as the amount of total energy radiated by unit area of its surface in unit time. Unit- $J\ m^{-2}s^{-1}$

Spectral emissive power E_λ of a body for the wavelength λ signifies the radiant energy per second per unit surface area per unit range of wavelength. Therefore, $E_\lambda d\lambda$ denotes the energy per unit area per second in the wavelength range between λ and $\lambda + d\lambda$. It is related to emissivity through the relation

hemispherical

$$\mathbf{E} = \int_0^\infty E_\lambda \, d\lambda$$

Emissivity of a surface → Ratio of the radiation emitted by the surface at a given temperature to the radiation emitted by a blackbody at the same temperature.

$$\varepsilon = \frac{E(T)}{E_b(T)} \qquad 0 \leq \varepsilon \leq 1$$

Emissivity of real surfaces = f(T, λ, direction of radiation)

Spectral absorptivity (a_λ) is defined as the fraction of incident energy absorbed per unit surface area per second at wavelength λ. Suppose that δQ_λ radiation of wavelength between λ and $\lambda + d\lambda$ is incident on a unit area of the surface of the body per second from all possible directions. If $a_\lambda \delta Q_\lambda$ is the amount of radiation absorbed, then a_λ signifies the absorptivity of the body for wavelength λ. a_λ has no dimensions;

Conservation of Radiant Energy:

Reflection, Absorption & Transmission

- Three things can happen when radiation with a given wavelength, λ, hits an object or substance:
  1. Part or all can be reflected:
     - fraction reflected: ______________________
     - This part does not interact with the object, it is reflected
  2. Part or all can be absorbed:
     - fraction absorbed: ______________________
     - This part is converted to another form of energy – usually heat energy, which raises the temperature of the object
  3. Part or all can be transmitted:
     - fraction transmitted: ______________________
     - This part does not interact with the object, it just goes through it.
- Since these are the only possibilities, it follows from the principle of conservation:

$$r_\lambda + a_\lambda + t_\lambda = 1$$

$$\text{absorptivity} = \frac{\text{absorbed radiation}}{\text{incident radiation}}$$

$$\text{reflectivity} = \frac{\text{reflected radiation}}{\text{incident radiation}}$$

$$\text{transmissivity} = \frac{\text{transmitted radiation}}{\text{incident radiation}}$$

Kirchhoff's Law: Relation Between e_λ and a_λ

The Kirchhoff's law states that *the ratio of the spectral emissive power* e_λ *to the spectral absorptivity* a_λ *for a particular wavelength* λ *is the same for all bodies at the same temperature and is equal to the emissive power of a perfectly black body at that temperature*. Mathematically, we write

$$\frac{e_\lambda}{a_\lambda} = E_\lambda$$

where E_λ is emissive power of a perfectly blackbody. Note that the ratio e_λ/a_λ is a universal function of λ and T.

Stefan-Boltzmann law

This law states that the energy radiated from a black body is proportional to the fourth power of the absolute temperature.

Wien Displacement Law Formula The Wien's Displacement Law provides the wavelength where the spectral radiance has maximum value. This law states that the black body radiation curve for different temperatures peaks at a wavelength invers proportional to the temperature.

Maximum wavelength = Wien's displacement constant / Temperature

The equation is:

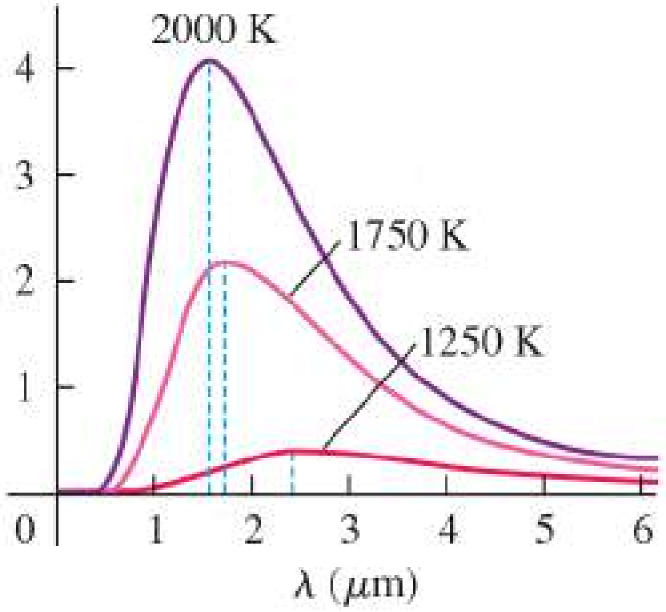
$\lambda_{max} = b/T$

Where:

λ_{max}: The peak of the wavelength

b: Wien's displacement constant. ($2.9*10^{(-3)}$ m K)

T: Absolute Temperature in Kelvin.



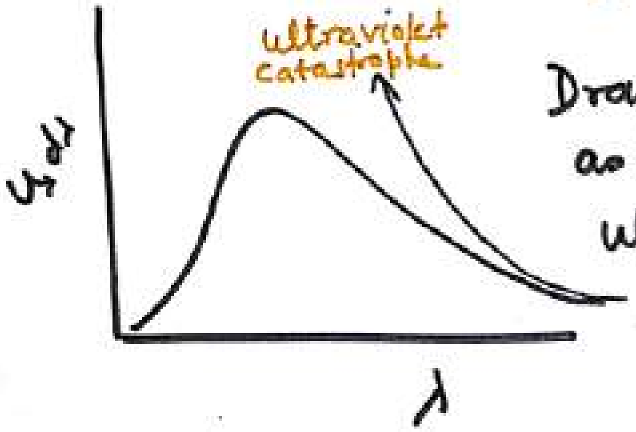
Drawbacks of Wien's Law: This law explains the energy distribution only in shorter wavelengths & fails to explain the energy distribution in longer wavelength region.

Weins law

$$u_\lambda d\lambda = \frac{c_1}{\lambda^5} e^{-c_2/\lambda T} d\lambda$$

Rayleigh Jeans – B. B radiation in enclosure consists of a no. of em waves, travelling in all directions, hit against walls, undergo multiple reflections & superimpose to form standing wave.

$$u_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$



Drawback → as $\lambda \to 0$, $u_\lambda \to \infty$ ultraviolet catastrophe

$u_\lambda d\lambda$ = No. of modes in wavelength rang λ to $\lambda + d\lambda$ X ave. energy of modes

$$= \frac{8\pi}{\lambda^4} d\lambda \times kT$$

Planck Radiation Law – (1900)

* Black body chamber is filled up not only with radiations but also with simple harmonic oscillators of molecular dimensions

* The oscillator in cavity walls could not have a continuous distribution of possible energies ϵ, but must have only specific energies

$$\epsilon_n = nh\nu \qquad n = 0, 1, 2 \ldots$$

* An oscillator emits radiation of frequency ν when it drops from one energy state (E_2) to the next lower one (E_1) and it absorbs radiation of frequency ν while going to the higher energy state (ie $E_1 \rightarrow E_2$)

* Each discrete bundle of energy is called a quantum.

* With oscillator energy limited to $nh\nu$, the average energy per oscillator in cavity walls and so per standing waves (as in R. Jeans) turns out not equal to kT but different from it.

* According to Boltzmann distribution the probability of a mode with energy ϵ at a given temp T is $e^{-\epsilon/kT}$

The av. energy of a mode is

$$\bar{\epsilon} = \frac{\sum_{n=0}^{\infty} nh\nu \, e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}}$$

$$\bar{\epsilon} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

So $u_\nu d\nu = \frac{8\pi\nu^2}{c^3} \times \frac{h\nu\, d\nu}{e^{h\nu/kT} - 1}$

$$\boxed{U_\nu d\nu = \frac{8\pi h \nu^3}{c^3} \frac{d\nu}{e^{h\nu/kT} - 1}}$$ Planck Radiation law

In terms of wavelength

$$\nu = \frac{c}{\lambda} \qquad d\nu = \left| -\frac{c}{\lambda^2} \right| d\lambda$$

$$U_\lambda d\lambda = \frac{8\pi h\, c^3}{\lambda^3 \times c^3} \times \frac{c}{\lambda^2} d\lambda \frac{1}{e^{hc/\lambda kT} - 1}$$

$$\boxed{U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT} - 1}}$$

As $\lambda \to 0$ $\quad e^{hc/\lambda kT} \to \infty$ & $U_\lambda \to 0$

i.e. No ultraviolet catastrophe

Show that Planck radiation law reduces to R. J. law for longer wavelengths. $\lambda \gg \frac{hc}{kT}$

$$e^{hc/\lambda kT} = 1 + \frac{hc}{\lambda kT} + \cdots \text{ higher order terms}$$

$\because \lambda$ is very large $\quad e^{hc/\lambda kT} \simeq 1 + \frac{hc}{\lambda kT}$

$$\Rightarrow U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{hc/\lambda kT}$$

$$\boxed{U_\lambda d\lambda = \frac{8\pi}{\lambda^4} kT\, d\lambda}$$ which is R. J. law.

Show that for $\lambda \ll \frac{hc}{kT}$ Planck radiation law reduces to Weins' distribution law.

If $\lambda \ll \frac{hc}{kT}$ $e^{hc/\lambda kT} \gg 1$

$$\Rightarrow \quad u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT}}$$

$$= \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda kT} d\lambda$$

$$u_\lambda d\lambda = \frac{C_1}{\lambda^5} e^{-C_2/\lambda T} d\lambda$$

Which is Weins law.

Calculation of total energy density

Inside a black body total energy density

$$u = \int_0^\infty u_\lambda \, d\lambda$$

using Planck radiation law

$$u = \int_0^\infty \frac{8\pi hc}{\lambda^5 (e^{hc/\lambda kT} - 1)} \, d\lambda$$

Let $\frac{hc}{\lambda kT} = x$

$$\Rightarrow \lambda = \frac{hc}{xkT} \qquad \Rightarrow d\lambda = \frac{-hc}{kTx^2} \, dx$$

$$u = \int_\infty^0 \frac{8\pi hc \, (xkT)^5}{(hc)^5 \, (e^x - 1)} \times \frac{-hc}{kTx^2} \, dx$$

$$= \int_0^\infty \frac{8\pi k^4 T^4}{h^3 c^3} \frac{x^3}{e^x - 1} \, dx$$

$$u = \frac{8\pi k^4 T^4}{h^3 c^3} \underbrace{\int_0^\infty \frac{x^3}{e^x - 1} \, dx}_{= \frac{\pi^4}{15}}$$

$$u = \left[\frac{8\pi^5 k^4}{15 h^3 c^3} \right] T^4$$

Deduction of Stefan Boltzmann's law

Emissive power

The emissive power i.e. the energy radiated per second by unit surface area of the blackbody is

$$E = \frac{\mathbf{c}\,\mathbf{u}}{4} \qquad \mathbf{u} = \left[\frac{8\pi^5 k^4}{15h^3c^3}\right]T^4$$

$$= \frac{2\pi^5 k^4}{15h^3c^2}T^4$$

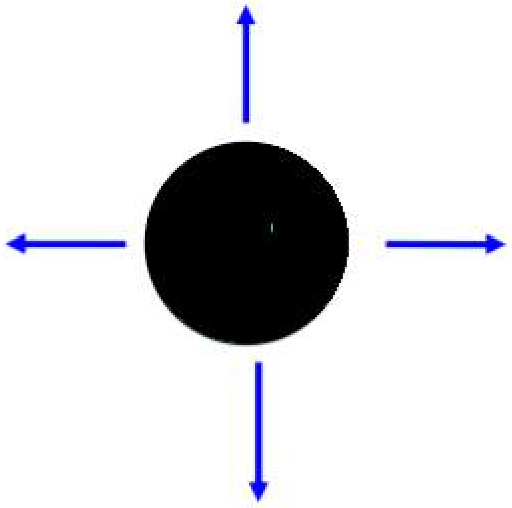
$$E = \sigma T^4$$

Which is Stefan's law. Where

$$\sigma = \frac{2\pi^5 k^4}{15h^3c^2}$$

The value of the Stefan-Boltzmann constant is approximately 5.67 x 10 $^{-8}$ watt per meter squared per kelvin to the fourth (W · m $^{-2}$ · K $^{-4}$).

If the radiation emitted normal to the surface and the energy density of radiation is u, then emissive power of the surface E=c u

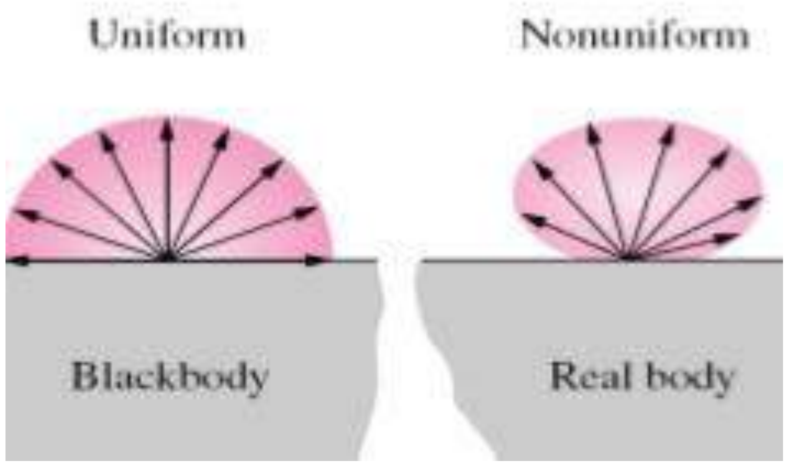


If the radiation is diffuse

Emitted uniformly in all directions

$$E=\frac{1}{4}cu$$

Integration over all angles provides a factor of ¼:



Blackbody is a diffuse emitter since it emits radiation energy uniformly in all directions.

Thermal radiation exerts pressure on the surface on which they are Incident.
If the intensity of directed beam of radiations incident normally to
The surface is I

Then Pressure P=u=$\frac{I}{c}$

If the radiation is diffused

P= $\frac{1}{3}\ u$

Deduction of Wien's Displacement Law

Planck's distribution law is

$$u_\lambda = \frac{8\pi hc}{\lambda^5(e^{hc/\lambda kT} - 1)}$$

u_λ is maximum at $\lambda = \lambda_m$ then

$$\left[\frac{du_\lambda}{d\lambda}\right]_{\lambda_m} = 0$$

This gives

at $\lambda = \lambda_m$

$$5 = \frac{ch}{\lambda kT}\frac{e^{hc/\lambda kT}}{e^{hc/\lambda kT} - 1}$$

at $\lambda = \lambda_m$
let $x = \frac{hc}{\lambda kT}$ then above equation reduces to

$$e^x = \frac{5}{5 - x}$$

or

$$x = ln5 - ln(5 - x)$$

This is a non algebric equation having solution

$$x \approx 4.965$$

Hence $\frac{hc}{\lambda_m kT} = 4.965$ or

$$\lambda_m T = \frac{hc}{k(4.965)}$$

Substituting the values of $h, c \& k$

$$\lambda_m T = 2.989 * 10^{-3}$$

Which is Wien's displacement law.

Thermodynamics of black body radiation

Let us make a black body enclosure with a piston, so that work may be done on or extracted from the radiation.

We have from Ist law of thermodynamics

$$\delta Q = dU + PdV$$

IInd law of thermodynamics $dS = \frac{\delta Q}{T}$

Now internal energy of radiation given by

$U = uV$ $\quad\quad$ V = Volume

Pressure of radiation $P = \frac{u}{3}$

$$\delta Q = d(uV) + \frac{1}{3} u\, dV$$

$$\Rightarrow \quad = \underline{u dV} + V du + \underline{\tfrac{1}{3} u dV}$$

$$\delta Q = V du + \frac{4}{3} u\, dV$$

$$\delta Q = T dS$$

$$T dS = V du + \frac{4}{3} u\, dV$$

$$dS = \frac{V}{T} du + \frac{4}{3} \frac{u}{T} dV$$

$\because$ dS is exact differential

$$dS = \left(\frac{\partial S}{\partial u}\right)_V du + \left(\frac{\partial S}{\partial V}\right)_u dV$$

$$ds = \left(\frac{\partial s}{\partial u}\right)_V du + \left(\frac{\partial s}{\partial V}\right)_u dV$$

We also have $ds = \left(\frac{V}{T}\right) du + \frac{4}{3}\frac{u}{T} dV$

Comparing
$$\left(\frac{\partial s}{\partial u}\right)_V = \frac{V}{T}$$

& $$\left(\frac{\partial s}{\partial V}\right)_u = \frac{4}{3}\frac{u}{T}$$

also $$\frac{\partial}{\partial V}\left(\frac{\partial s}{\partial u}\right)_V = \frac{\partial}{\partial u}\left(\frac{\partial s}{\partial V}\right)_u$$

$$\Rightarrow \frac{\partial}{\partial V}\left(\frac{V}{T}\right) = \frac{\partial}{\partial u}\left(\frac{4}{3}\frac{u}{T}\right)$$

$$\frac{1}{T} = \frac{4}{3}\frac{1}{T} - \frac{4}{3}\frac{u}{T^2}\frac{\partial T}{\partial u}$$

$$\frac{4}{3}\frac{u}{T^2}\frac{\partial T}{\partial u} = \frac{4}{3T} - \frac{1}{T}$$

$$\frac{\partial T}{\partial u} = \frac{T}{4u}$$

$$4\frac{\partial T}{T} = \frac{\partial u}{u}$$

$$\log_e u = 4\log_e T + \text{const} \rightarrow \log_e a$$

$$\Rightarrow \log_e u = \log_e aT^4$$

or $\boxed{u = aT^4}$

which is another form of Stefan's law

Expression for entropy, pressure, F & G of the b.b. radiation

$$\frac{\partial S}{\partial V} = \frac{4}{3}\frac{u}{T}$$

$$\frac{\partial S}{\partial V} = \frac{4}{3}\frac{aT^4}{T}$$

$$\frac{\partial S}{\partial V} = \frac{4}{3}aT^3$$

$$\Rightarrow \boxed{S = \frac{4}{3}aT^3V}$$

$$P = \frac{u}{3} = \frac{1}{3}aT^4$$

$$\boxed{P = \frac{1}{3}aT^4}$$

$$F = U - TS$$

$$= aVT^4 - \frac{4}{3}aVT^4$$

$$\boxed{F = -\frac{1}{3}aVT^4}$$

$$G = F + PV$$

$$= -\frac{1}{3}aVT^4 + \frac{1}{3}aT^4V$$

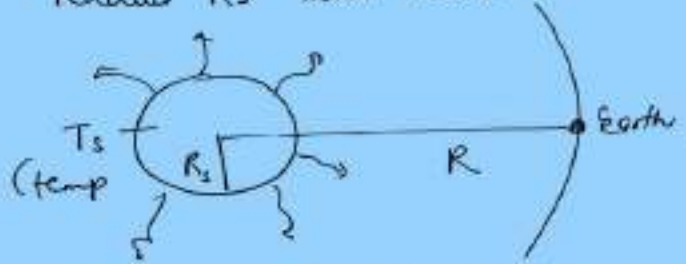
$$\boxed{G = 0}$$

Solar constant - Solar energy received on earth surface per unit area per unit time perpendicular to the sun's rays.

unit → $\frac{J}{sec\, m^2}$ or $\frac{W}{m^2}$

Let us assume sun to be a perfect B.B with e = 1

Radius R_s and temp T_s



Sun is emitting solar energy uniformly in all the directions

a small fraction of this solar energy will be received by Earth at a distance R from the sun.

Thermal power emitted by sun $P_s = \sigma T_s^4 (4\pi R_s^2)$

Power received per unit area on Earth ⇒ Solar constant (S)

$$S = \frac{P_s}{4\pi R^2}$$

$$= \frac{\sigma T_s^4 (4\pi R_s^2)}{4\pi R^2}$$

$$\boxed{S = \sigma T_s^4 \left(\frac{R_s}{R}\right)^2}$$

The **value** of the **constant** is approximately 1.366 kilowatts per square metre.

Isothermal Expansion of blackbody radiation

Consider reversible isothermal expansion of a volume V of blackbody radiation, by small amount dV, at constant temp T.

To calculate the amount of heat dQ supplied to maintain the constant temperature can be calculated using TdS equation

$$TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$$

$dT = 0$

$$\Rightarrow TdS = T\frac{dQ}{T}$$

or

$$dQ = T\left(\frac{\partial S}{\partial V}\right)_T dV$$

We have seen that $\frac{\partial S}{\partial V} = \frac{4}{3} a T^3$

$$(dQ)_T = T \times \frac{4}{3} a T^3 dV$$

$$(dQ)_T = \frac{4}{3} aT^4 dV$$

with change in volume internal energy also changes

$\rightarrow$ $$U = uV$$
$$= aT^4 V$$

$$dU = aT^4 dV$$

The work done in dV expansion at constant temp $dW = P dV$

$$= \frac{1}{3} aT^4 dV$$

Now $dW + dU = \frac{1}{3} a T^4 dV + aT^4 dV$

$$dW + dU = \frac{4}{3} aT^4 dV$$

& $$dQ = \frac{4}{3} aT^4 dV$$

or $$dQ = dU + dW$$

Which implies the system follows I[st] law of thermodynamics.

For isothermal compression dV, dW & dU all will have negative sign & hence I[st] law will be followed.

Reversible adiabatic expansion

In rev. adiabatic process entropy S = constant

The entropy of black body radiation

$$S = \frac{4}{3} a V T^3$$

$a \Rightarrow$ radiation density constant

$$a = \frac{4\sigma}{c} = 7.56 \times 10^{-16}\ J K^{-4} m^{-3}$$

$$\sigma = \frac{ac}{4} = 5.6704 \times 10^{-8}\ J s^{-1} m^{-2} K^{-4}$$

$$S = \frac{4}{3} a V T^3$$

$$\Rightarrow T^3 = \left(\frac{3S}{4aV}\right)$$

$$T = \left(\frac{3S}{4aV}\right)^{\frac{1}{3}}$$

$$T = \left(\frac{3S}{4aV}\right)^{\frac{1}{3}}$$

& $$P = \frac{1}{3} a T^4$$

$$= \frac{1}{3} a \left(\frac{3S}{4aV}\right)^{\frac{4}{3}}$$

$$P = [\quad] V^{-\frac{4}{3}}$$

or $$\boxed{PV^{\frac{4}{3}} = \text{constant}}$$

$\Rightarrow$ In an adiabatic process taking place in black body radiation the quantity $PV^{4/3}$ remains constant.

Comparing with PV^{γ} in case of an ideal gas

$$\Rightarrow \gamma = \frac{C_p}{C_v} = \frac{4}{3} \text{ for blackbody radiation}$$

In case of blackbody radiation the work done in reversible adiabatic expansion of blackbody radiation from initial state P_i & V_i to final state P_f & V_f

$$W = \int_{P_i, V_i}^{P_f, V_f} P\,dV$$

$$\boxed{W = 3\left[P_f V_f - P_i V_i\right]}$$

When the **emissivity** of non-black surface **is constant at all temperatures and throughout the entire range of wavelength**, the surface is called Gray Body.

PROBLEMS

1. The temperature of a person's skin is 35^0 C. Calculate (a)The wavelength at which the radiation emitted from the skin reaches its peak (b) The net loss of power by body in the room at 20^0 C, take emittance of skin to be 0.98 and surface area of a typical person can be taken as 2 m^2 . (c) estimate net loss of energy during one day in kcal/sec?

2. The earth receives solar radiation at a rate of 8.2 J cm-2 min-1. Assuming that the sun radiates like a black body, calculate the surface temperature of the sun. The angle subtended by the sun on the earth is 0.53^0 .

3. Calculate the average energy of an oscillator of frequency 0.6 x 10^{14} sec-1 at the temperature of 1500 K, when it is
(i) A classical oscillator (ii) a Planck oscillator

4. Calculate the number of modes in the frequency range from 5000 to 5001 Ǻ in an enclosure of volume 100 cm^2.

5. A body at temperature 1500 K radiates out maximum energy at the wavelength 20,000 Ǻ. If the sun radiates out maximum energy at 5000 Ǻ, calculate the temperature of the sun.

6. The filament of a light bulb is cylindrical with length l=20 m.m. and radius r=0.05 mm. The filament is maintained at a temperature T = 5000 K by an electric current. The filament behaves as a black body, emitting radiation isotopically. At night you observe the light bulb from a distance of 10 km with pupil of your eye fully dilated to the radius 3 mm. (a) What is the total power emitted by the filament? (b)How much radiation power enters your eye? (c) How many photons enter your eye every second? You can assume the average wavelength for the radiation is 600 nm.