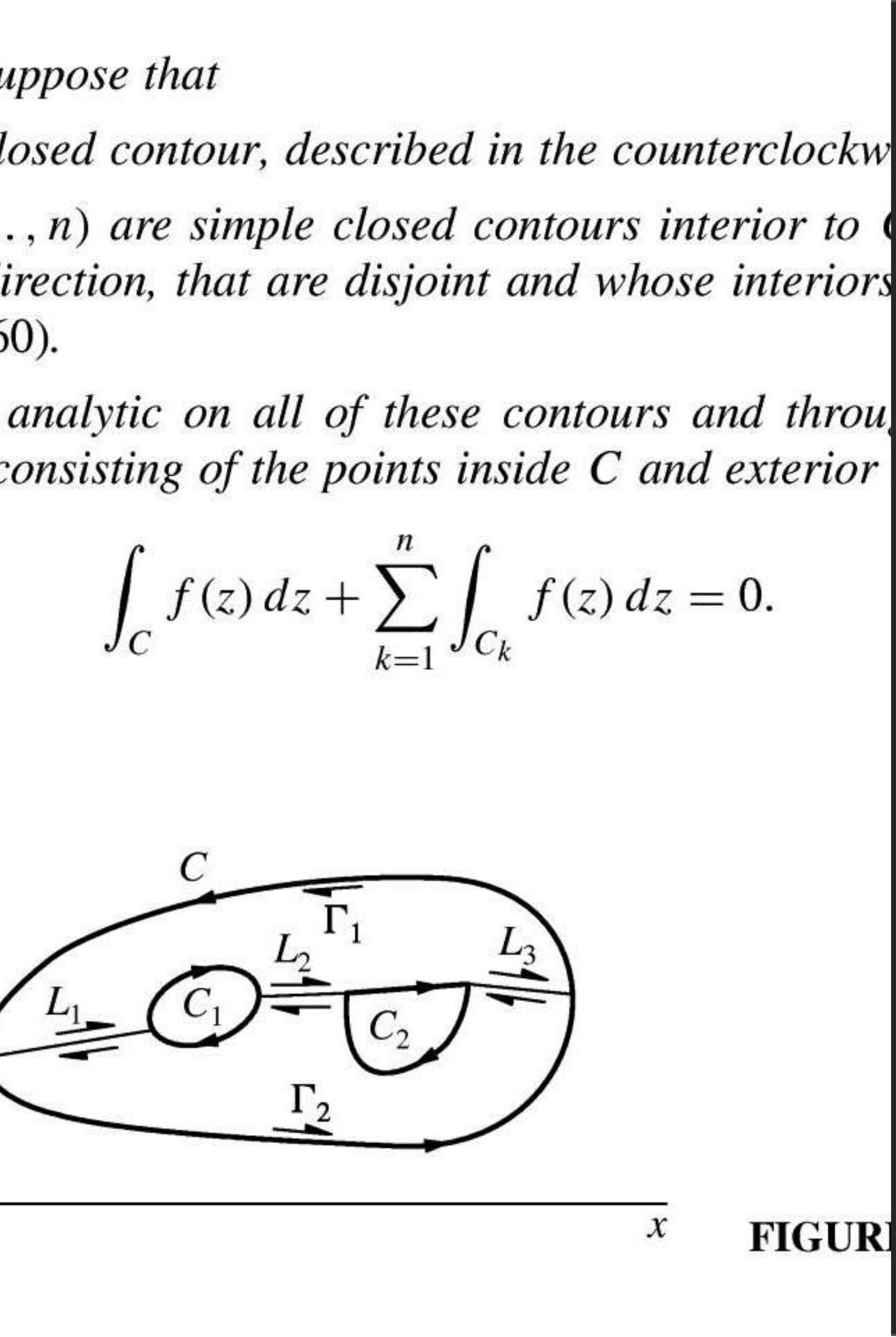

uppose that

losed contour, described in the counterclockw

.., n) are simple closed contours interior to (

irection, that are disjoint and whose interiors

50).

analytic on all of these contours and throu

consisting of the points inside C and exterior

$$\int_C f(z)\,dz + \sum_{k=1}^{n} \int_{C_k} f(z)\,dz = 0.$$

Cauchy Riemann Equation, Complex Analysis

An exploration of complex analysis, including key concepts and the fundamental Cauchy Riemann Equation. Discover its applications and implications in various fields.

KB by Kajal Bamotra

Introduction to Complex Analysis

Complex analysis is the branch of mathematics that extends the ideas and methods of real analysis to complex numbers. It studies functions that depend on complex variables.

Key Concepts in Complex Analysis

Complex Numbers

Numbers in the form a + bi, where a and b are real numbers and i is the imaginary unit.

Analytic Functions

Functions that are differentiable at every point in their domain, represented by power series.

Contour Integrals

Integrals taken along curves in the complex plane, used to evaluate complex integrals.

Residue Theory

Calculating complex integrals using the residue of a function at its singularities.

Cauchy Riemann Equation

The Cauchy Riemann Equation is a set of necessary conditions for a complex-valued function to be analytic, expressing the relationship between the real and imaginary parts of the function.

1. **Definition and Basic Properties**

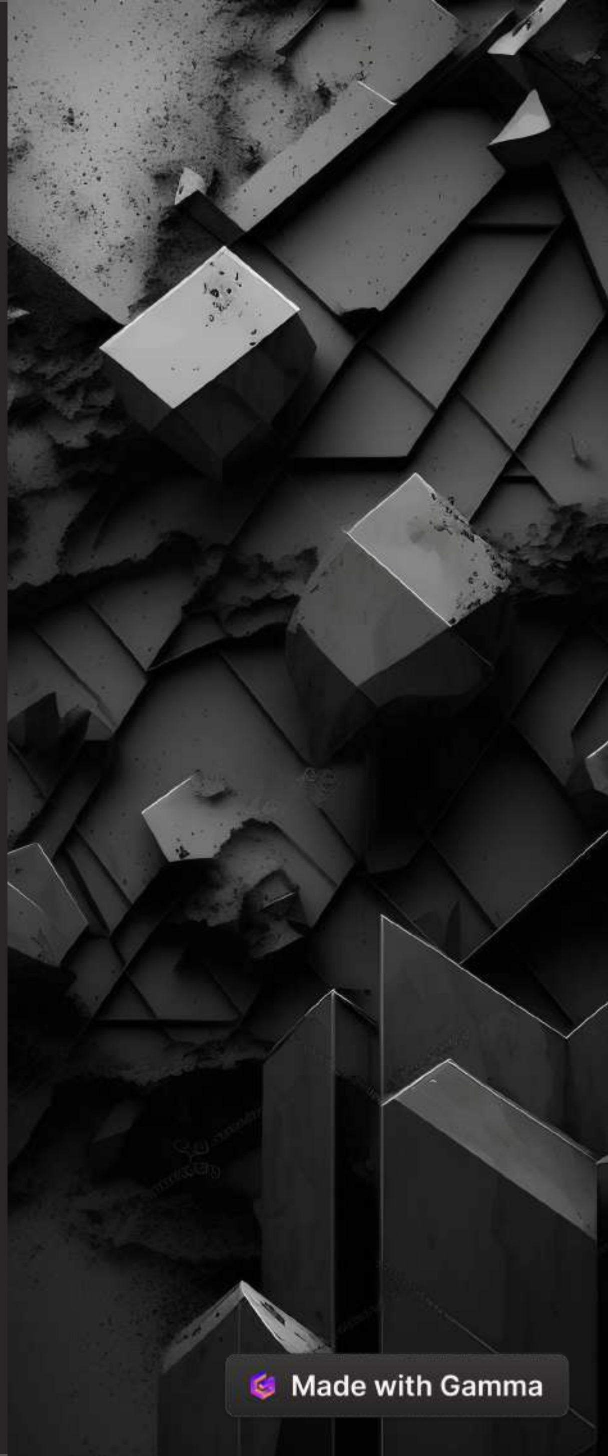
   Derive the Cauchy Riemann Equation from the definition of complex differentiability and explore its foundational properties.

2. **Derivation and Implications**

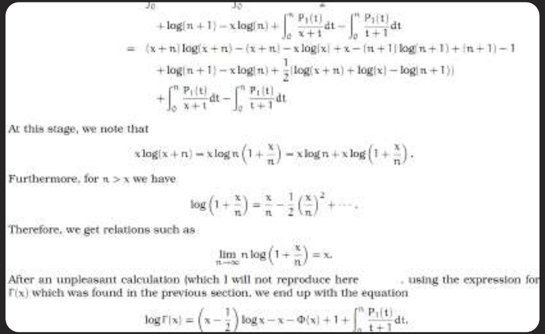
   Uncover the derivation of the Cauchy Riemann Equation and understand its implications for the behavior of analytic functions.

3. **Applications of the Cauchy Riemann Equation**

   Discover the practical applications of the Cauchy Riemann Equation in various fields of study, from physics to engineering.



Applications of the Cauchy Riemann Equation



$$+\log(n+1) - x\log(n) + \int_0^n \frac{P_1(t)}{x+t}dt - \int_0^n \frac{P_1(t)}{t+1}dt$$

$$= (x+n)\log(x+n) - (x+n) - x\log(x) + x - (n+1)\log(n+1) + (n+1) - 1$$
$$+\log(n+1) - x\log(n) + \frac{1}{2}(\log(x+n) + \log(x) - \log(n+1))$$
$$+\int_0^n \frac{P_1(t)}{x+t}dt - \int_0^n \frac{P_1(t)}{t+1}dt$$

At this stage, we note that

$$x\log(x+n) = x\log n\left(1+\frac{x}{n}\right) = x\log n + x\log\left(1+\frac{x}{n}\right).$$

Furthermore, for $n > x$ we have

$$\log\left(1+\frac{x}{n}\right) = \frac{x}{n} - \frac{1}{2}\left(\frac{x}{n}\right)^2 + \cdots.$$

Therefore, we get relations such as

$$\lim_{n\to\infty} n\log\left(1+\frac{x}{n}\right) = x.$$

After an unpleasant calculation (which I will not reproduce here , using the expression for $\Gamma(x)$ which was found in the previous section, we end up with the equation

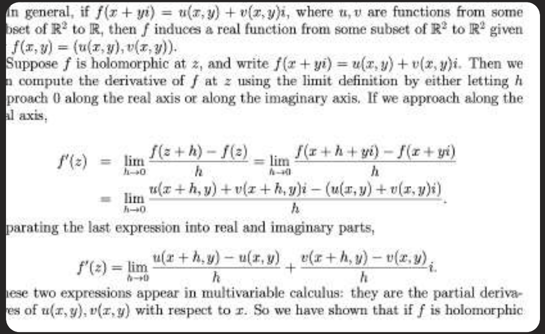
$$\log\Gamma(x) = \left(x-\frac{1}{2}\right)\log x - x - \Phi(x) + 1 + \int_0^n \frac{P_1(t)}{t+1}dt,$$

Analytic Functions

Explore the role of the Cauchy Riemann Equation in characterizing analytic functions and their behavior in physics.

Harmonic Functions

Investigate how the Cauchy Riemann Equation leads to harmonic functions, which have diverse applications in engineering.



In general, if $f(x+yi) = u(x,y) + v(x,y)i$, where u, v are functions from some bset of $\mathbb{R}^2$ to $\mathbb{R}$, then f induces a real function from some subset of $\mathbb{R}^2$ to $\mathbb{R}^2$ given $f(x,y) = (u(x,y), v(x,y))$.

Suppose f is holomorphic at z, and write $f(x+yi) = u(x,y) + v(x,y)i$. Then we n compute the derivative of f at z using the limit definition by either letting h proach 0 along the real axis or along the imaginary axis. If we approach along the l axis,

$$f'(z) = \lim_{h\to0}\frac{f(z+h)-f(z)}{h} = \lim_{h\to0}\frac{f(x+h+yi)-f(x+yi)}{h}$$
$$= \lim_{h\to0}\frac{u(x+h,y)+v(x+h,y)i-(u(x,y)+v(x,y)i)}{h}.$$

parating the last expression into real and imaginary parts,

$$f'(z) = \lim_{h\to0}\frac{u(x+h,y)-u(x,y)}{h} + \frac{v(x+h,y)-v(x,y)}{h}i.$$

iese two expressions appear in multivariable calculus: they are the partial deriva- es of $u(x,y), v(x,y)$ with respect to x. So we have shown that if f is holomorphic

Consequences for Differentiability

Examine the implications of the Cauchy Riemann Equation for the differentiability of complex functions.

Conclusion and Further Study Suggestions

Complex analysis and the Cauchy Riemann Equation provide a powerful framework for understanding functions that depend on complex variables. Enhance your understanding by exploring related topics such as complex integration and residue theory.