Replacement Problems

Replacement problems can be classified into following categories:

(a) When the equipment / assets deteriorates with time

And the value of the ~~asset~~ money:

(i) Does not change with time.

(ii) Changes with time.

(b) When the items fails suddenly.

Replacement Policy when value of money does not change with time:

In this case, we determine the optimum replacement age of an equipment whose running cost or maintenance cost increases with time but the value of money does not change.

Let C denotes Capital cost of equipment.

S denotes Scrap value.

n denotes no. of years the equipment would be in use

$f(t)$ denotes maintenance cost function

$A(n)$ denotes Average total annual cost

Case I: When t is continuous → If the equipment is used n years, the total cost incurred during this period is given by

Total cost = Capital cost − Scrap value + Maintenance cost

$$= C - S + \int_0^n f(t)\,dt$$

$\therefore$ Average annual cost is

$$A(n) = \frac{1}{n}(\text{Total cost}) = \frac{C-S}{n} + \frac{1}{n}\int_0^n f(t)\,dt$$

For minimum value,

$$\frac{d}{dn}A(n) = 0 \Rightarrow \frac{-(C-S)}{n^2} - \frac{1}{n^2}\int_0^n f(t)\,dt + \frac{1}{n}f(n) = 0$$

$$\Rightarrow f(n) = \frac{C-S}{n} + \frac{1}{n}\int_0^n f(t)\,dt$$

$$= A(n)$$

Also $\frac{d^2}{dn^2}A(n) > 0$ at $f(n) = A(n)$

Thus we conclude that the equipment should be replaced when the maintenance becomes equal to average total cost.

Case II : When t is discrete variable :→

Here n & t has values $1, 2, 3, \ldots$

$$A(n) = \frac{C-S}{n} + \frac{1}{n}\sum_{t=1}^{n} f(t)$$

Now, $A(n)$ will be minimum for that value of n for which $A(n+1) \geq A(n)$ & $A(n-1) \geq A(n)$

$\Rightarrow A(n-1) - A(n) \geq 0$ & $A(n) - A(n-1) \leq 0$

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Now, $A(n+1) = \frac{C-S}{n+1} + \frac{1}{n+1} \sum_{t=1}^{n+1} f(t)$.

$$= \frac{1}{n+1}\left(C-S + \sum_{t=1}^{n} f(t)\right) + \frac{1}{n+1} f(n+1)$$

$$= \frac{n A(n)}{n+1} + \frac{1}{n+1} f(n+1)$$

$$\therefore A(n+1) - A(n) = \frac{1}{n+1}\left[n\{A(n) + f(n+1)\}\right] - A(n)$$

$$= \frac{1}{n+1}\left[f(n+1) - A(n)\right]$$

$$\therefore A(n+1) - A(n) \geq 0 \Rightarrow \frac{1}{n+1} f(n+1) - A(n) \geq 0$$

$$\Rightarrow \boxed{f(n+1) \geq A(n)}$$

Illy $A(n) - (An-1) \leq 0 \Rightarrow \boxed{f(n) \leq A(n-1)}$

Thus the optimum policy suggest that replace the equipment at end of n years if the maintenance cost for $(n+1)^{th}$ year is more than the avg. annual cost of n^{th} year, and, the n^{th} year's maintenance cost is less than the previous year's avg total annual cost.