Characteristics of the model

1. $E(n) = \sum_{n=0}^{N} n P_n$

$= P_0 \sum_{n=0}^{N} n \rho^n$

$= P_0 \rho \sum_{n=0}^{N} \frac{d}{d\rho} \rho^n$

$= P_0 \rho \frac{d}{d\rho} \sum_{n=0}^{N} \rho^n$

$= P_0 \rho \frac{d}{d\rho} \left(\frac{1-\rho^{N+1}}{1-\rho} \right)$ [If $\rho = 1$ then $\lambda = \mu$ (no queue

$= P_0 \rho \left[\frac{(1-\rho)\left(-(N+1)\rho^N\right) + 1-\rho^{N+1}}{(1-\rho)^2} \right]$

$= P_0 \rho \left[\frac{-(N+1)(1-\rho)\rho^N + (1-\rho^{N+1}}{(1-\rho)^2} \right.$

$$= P_0 \rho \left[\frac{[(N+1) + (N+1)\rho]\rho^N + (1-\rho^{N+1})}{(1-\rho)^2} \right]$$

$$= P_0 \rho \left[\frac{-N\rho^N - \rho^N + N\rho^{N+1} + \cancel{\rho^{N+1}} + 1 - \cancel{\rho^{N+1}}}{(1-\rho)^2} \right]$$

$$= \frac{\rho(-N\rho^N - \rho^N + N\rho^{N+1} + 1)}{(1-\rho)(1-\rho^{N+1})}$$

$$= \frac{\rho[1-(N+1)\rho^N + N\rho^{N+1}]}{(1-\rho)(1-\rho^{N+1})}$$

2. $E(m) = \sum_{m=0}^{N} m P_n \qquad m = n-1$

$$= \sum_{n=1}^{N} (n-1) P_n$$

$$= \sum n P_n - \sum_{n=1}^{N} P_n$$

$$= E(n) - (1 - P_0)$$

$$= \frac{\rho[1-(N+1)\rho^N + N\rho^{N+1}]}{(1-\rho)(1-\rho^{N+1})} - \left[1 - \left(\frac{1-\rho}{1-\rho^{N+1}}\right)\right]$$

$$= \frac{\rho[1-(N+1)\rho^N + N\rho^{N+1}]}{(1-\rho)(1-\rho^{N+1})} - \left[\frac{\cancel{1} - \rho^{N+1} - \cancel{1} + \rho}{(1-\rho^{N+1})}\right]$$

$$= \frac{\rho[1-(N+1)\rho^N + N\rho^{N+1}] - \rho(1-\rho)(1-\rho^N)}{(1-\rho)(1-\rho^{N+1})}$$

$$= \frac{\rho\left[1-(N+1)\rho^{N}+N\rho^{N+1}\right]-\rho\left[1-\rho-\rho^{N}+\rho^{N+1}\right]}{(1-\rho)(1-\rho^{N+1})}$$

$$= \frac{\rho\left[\cancel{1}-(N+1)\rho^{N}+N\rho^{N+1}\cancel{-1}+\rho+\rho^{N}-\rho^{N+1}\right]}{(1-\rho)(1-\rho^{N+1})}$$

$$= \frac{\rho\left[\rho-N\rho^{N}+(N-1)\rho^{N+1}\right]}{(1-\rho)(1-\rho^{N+1})}$$

$$= \frac{\rho^{2}\left[1-N\rho^{N-1}+(N-1)\rho^{N}\right]}{(1-\rho)(1-\rho^{N+1})}$$

3. $E(v) = \frac{E(n)}{\lambda}$

4. $E(w) = E(v) - 1/\mu$