

Production Theory

Production

- The process of transformation of resources (like land, labour, capital and entrepreneurship) into goods and services of utility to consumers and/or producers.
- The process of creation of value or wealth through the production of goods and services that have economic value.
- Goods includes all tangible items such as furniture, house, machine, food, car, television etc
- Services include all intangible items, like banking, education, management, consultancy, transportation.

Types of Inputs

Technology

- determines the type, quantity and proportion of inputs.
- **Fixed and Variable Inputs**
- Production analysis of a firm uses two distinct time frames:
 - short run
 - long run
- *Variable input* : that can be made to vary in the short run, e.g. raw material, unskilled/semi skilled labour, etc.
- *Fixed input*: that cannot be varied in the short run, e.g. land, machine, technology, skill set, etc.

Factors of Production

5 factors of production

- **Land(I)**
 - gift of nature
 - Reward is called as rent
- **Labour(L)**
 - Skilled as well as unskilled.
 - Reward is called as wages
- **Capital(K)**
 - Wealth which is used for further production as machine/ equipment/intermediary good
 - Reward is called as interest
- **Enterprise(E)**
 - The ability and action to take risk of collecting, coordinating, and utilizing all the factors of production for the purpose of uncertain economic gains
 - Reward is called as profit
- **Organization(O)**
 - Combination of highly skilled labour and specialized human capital/managerial aspect of business.
 - Reward is called as salary

Production Function

- A technological relationship between physical inputs and physical outputs over a given period of time.
- production function is:
- Always related to a given time period
- Always related to a certain level of technology
- Depends upon relation between inputs.
- Normally a production function is written as:
 - $$Q = f (L,K,I,R,E)$$
 - where Q is the maximum quantity of output of a good being produced, and L=labour; K=capital; I=land; R=raw material; E= efficiency parameter.
- *Technical efficiency is defined as a situation when using more of one input with either the same amount or more of the other input must increase output.*

Production Function with One Variable Input

- short term production function
- Shows the maximum output a firm can produce when only one of its inputs can be varied, other inputs remaining fixed:

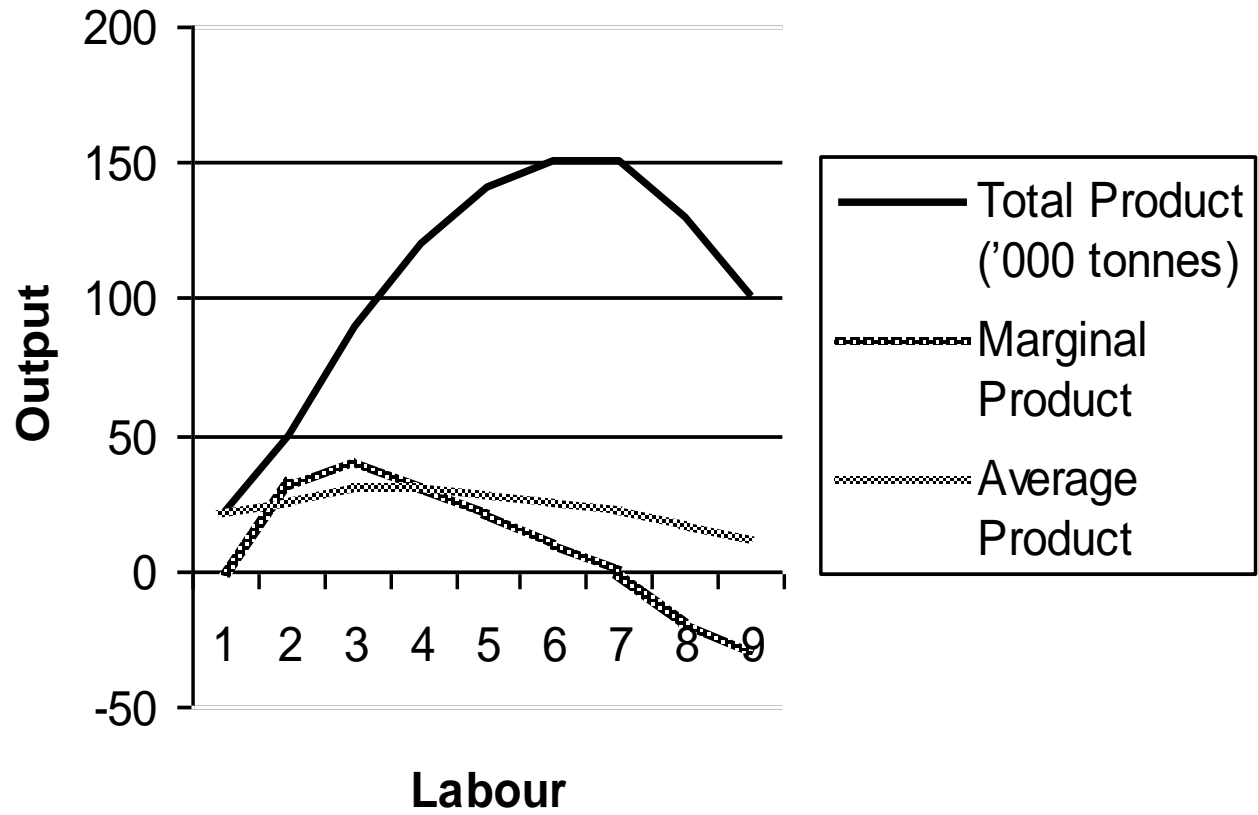
- $Q = f(L, \bar{K})$
- where Q = output, L = labour and K = fixed amount of capital

- Total product is a function of labour: $TP_L = f(\bar{K}, L)$

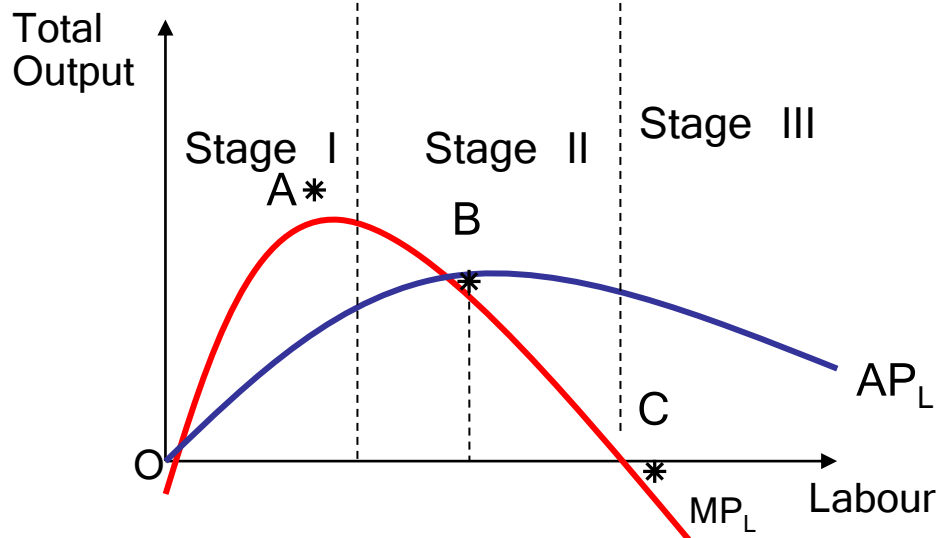
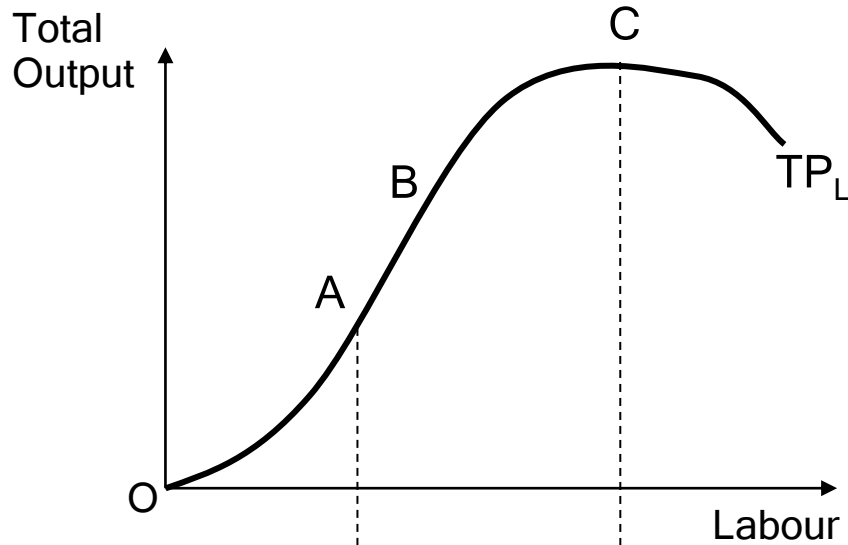
- Average Product (AP) = $AP_L = \frac{TP}{L}$

- Marginal product of labour (MP_L) = $MP_L = \frac{\Delta TP}{\Delta L}$

Labour ('00 units)	Total Product ('000 tonnes)	MP	AP	Stages
1	20	-	20	Increasing returns
2	50	30	25	
3	90	40	30	
4	120	30	30	Diminishing returns
5	140	20	28	
6	150	10	25	
7	150	0	21.5	
8	130	-20	16.3	Negative returns
9	100	-30	11.1	



Law of Variable Proportions

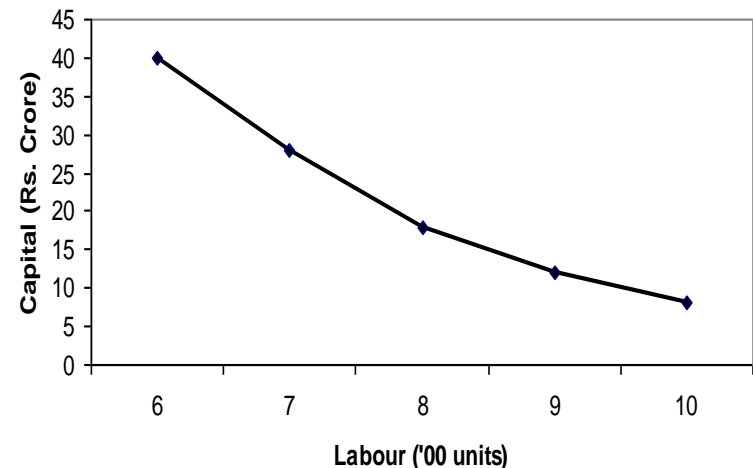


- **First stage**
 - Increasing Returns to the Variable Factor
 - $MP > 0$ and $MP > AP$
- **Second stage**
 - Diminishing Returns to a Variable Factor
 - $MP > 0$ and $MP < AP$
- **Third Stage**
 - Negative Returns
 - $MP < 0$ while AP is falling but positive
 - Technically inefficient stage of production
 - A rational firm will never operate in this stage

Production Function with Two Variable Inputs

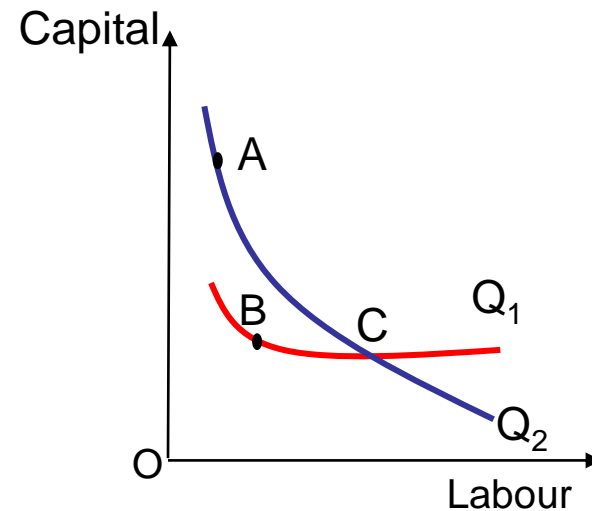
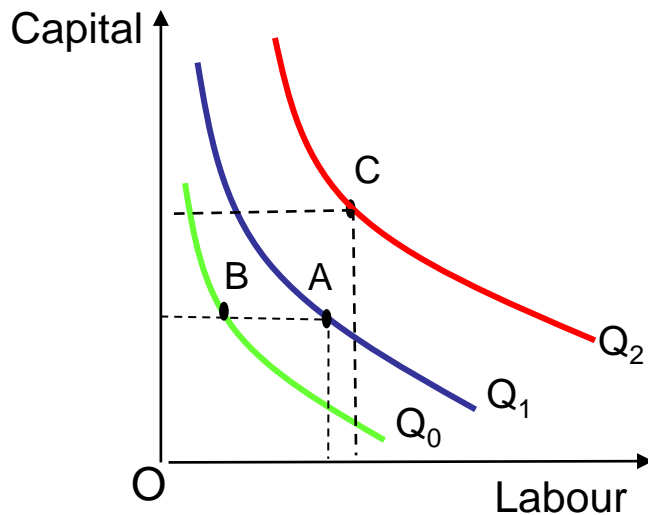
- All inputs are variable in long run and only two inputs are used
- Firm has the opportunity to select that combination of inputs which maximizes returns
- Curves showing such production function are called *isoquants* or *iso-product curves*.
- **An *isoquant* is the locus of all technically efficient combinations of two inputs for producing a given level of output**
- Represented as: $\bar{Q} = f(L, K)$

Capital (Rs. crore)	Labour ('00 units)
40	6
28	7
18	8
12	9
8	10



Characteristics of Isoquants

- Downward sloping
- Convex to the origin
- A higher isoquant represents a higher output
- Two isoquants can not intersect each other



Marginal Rate of Technical Substitution

- Measures the reduction in one input, due to unit increase in the other input that is just sufficient to maintain the same level of output.

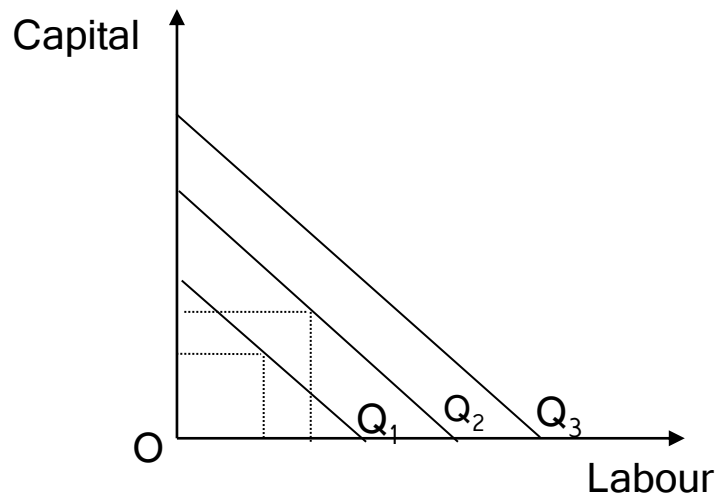
$$MRTS_{LK} = -\frac{\Delta K}{\Delta L}$$

- MRTS of labour for capital is equal to the slope of the isoquant.
- It is also equal to the ratio of the marginal product of one input to the marginal product of other input

$$\Delta Q = MP_L \times \Delta L + MP_K \times \Delta K$$

$$0 = MP_L \times \Delta L + MP_K \times \Delta K$$

$$MRTS_{LK} = \frac{MP_L}{MP_K} = -\frac{\Delta K}{\Delta L}$$



Linear isoquants

$$\bar{Q} = f(L, K) = \alpha K + \beta L$$

- Perfect substitutability between two factors
- Isoquants are downward sloping straight lines
- Constant MRTS

Elasticity of Substitution

- Measures the percentage change in factor proportions due to a change in marginal rate of technical substitution

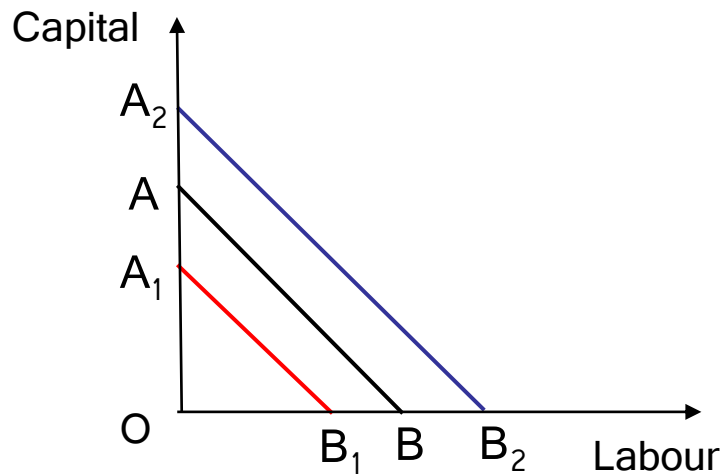
$$\sigma = \frac{d(K/L)/K/L}{d(MRTS)/MRTS}$$

- σ is effectively a measure of the curvature of an isoquant
- More curved or convex is the isoquant, the lower is σ
- In Leontief (zero substitution) technology, with L shaped isoquants, there is no substitutability between the inputs
 - $\sigma = 0$
- In perfect substitution or linear production technology, the MRTS does not change at all along the isoquant.
 - σ is infinite

Isocost Lines

Total Cost is sum of Labour cost (wL) and Capital cost (rK) where wage (w) and interest (r)

$$C = wL + rK$$

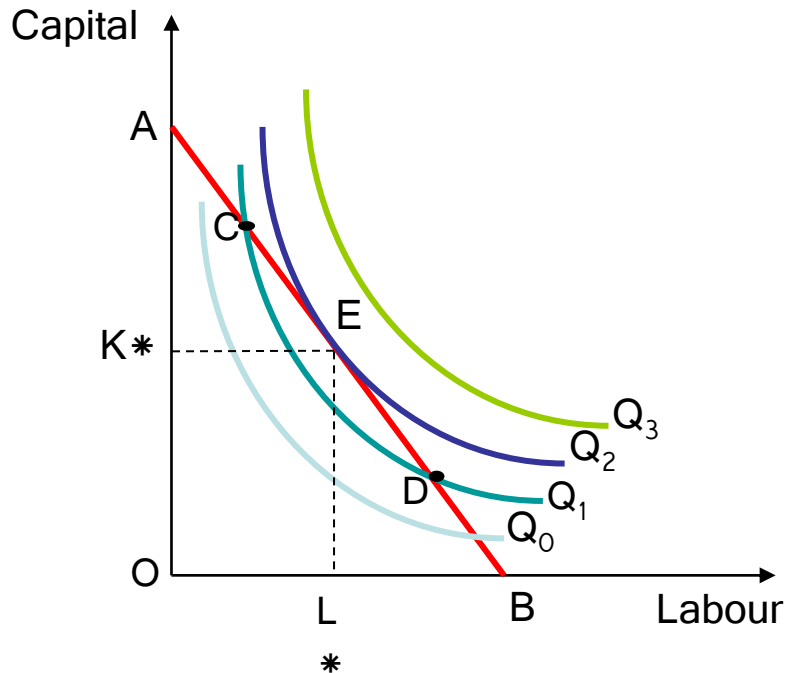


- The isocost line represents the locus of points of all the different combinations of two inputs that a firm can procure, given the total cost and prices of the inputs.

The (absolute) slope of this line is equal to the ratio of the input prices.

$$Slope = -\frac{\Delta K}{\Delta L} = \frac{C/r}{C/w} = \frac{w}{r}$$

Producer's Equilibrium

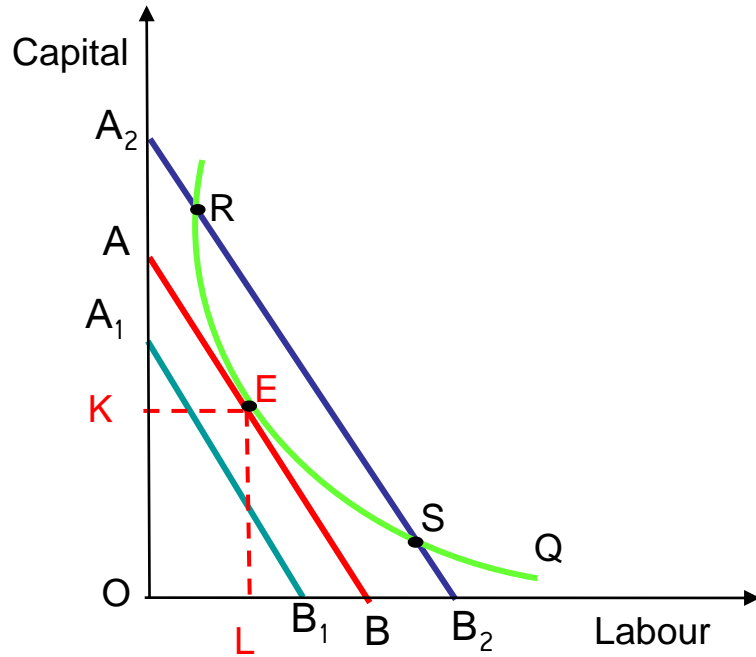


Maximization of output subject to cost constraint

Necessary condition for equilibrium
Slope of isoquant = Slope of isocost line

- AB is the isocost line
- Any point below AB is feasible but not desirable
- E is the point of tangency of Q_2 with isocost line AB
 - Corresponds to the highest level of output with given cost function.
- Firm would employ L^* and K^* units of labour and capital
- Q_3 is beyond reach of the firm
- Points C and D are also on the same isocost line, but they are on isoquant Q_1 , which is lower to Q_2 . Hence show lower output.
- E is preferred to C and D, which is on the highest *feasible* isoquant.

Producer's Equilibrium

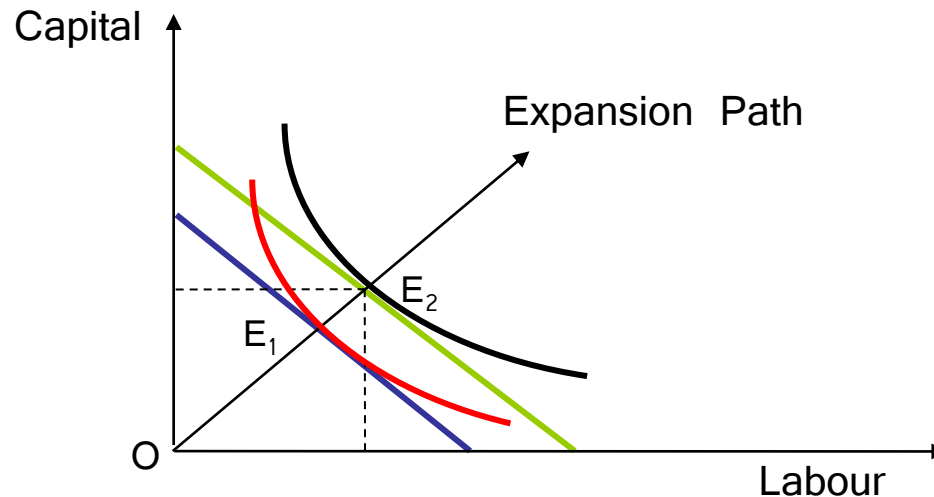


Minimization of cost for a given level of output

Necessary condition for equilibrium
Slope of isoquant = Slope of isocost line

- Firm has decided the level output to be produced shown by the isoquant Q
 - Will be indifferent between output combinations shown by R, S, E on isoquant Q.
- Has to ascertain that combination of inputs Labour and Capital which minimizes the cost of production
- Hence a map of isocost lines will be prepared
- The isocost lines are parallel to each other because price of the inputs is given.
- A_1B_1 line is not feasible
- It will use OK and OL of capital and labour respectively, at point E which is also on AB, the lowest possible isocost line.
- R, S are not desirable because they are on higher cost line A_2B_2 .

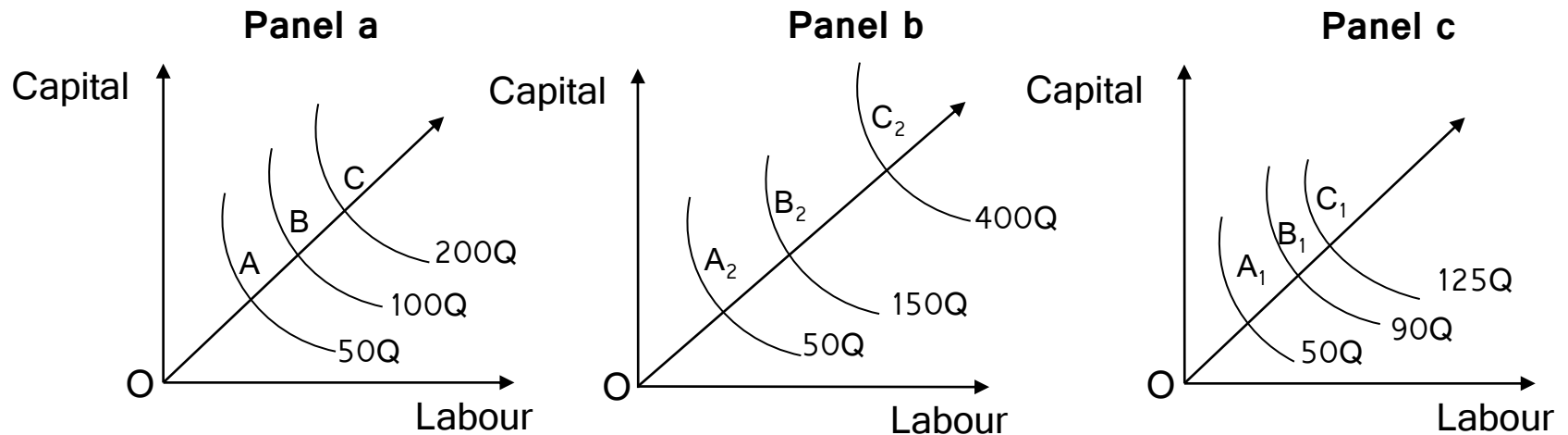
Expansion Path



- Line formed by joining the tangency points between various isocost lines and the corresponding highest attainable isoquants is known as Expansion Path.
- For homogeneous production function and given factor prices (and hence factor ratio):
 - expansion path is a straight line through the origin.
- For non-homogeneous production function:
 - optimal expansion path is non linear.

Returns to Scale

- Returns to Scale show the degree by which the level of output changes in response to a given change in *all* the inputs in a production system.



- **Constant Returns to Scale** : When a proportional increase in all inputs yields an equal proportional increase in output (*Panel a*)
- **Increasing Returns to Scale** : When a proportional increase in all inputs yields a more than proportional increase in output (*Panel b*).
- **Decreasing Returns to Scale** : When a proportional increase in all inputs yields a less than proportional increase in output (*Panel c*).

Cobb-Douglas Production Function

- Proposed by Wicksell and tested against statistical evidence by Charles W. Cobb and Paul H. Douglas in 1928
- $Q = AK^\alpha L^\beta$ where α , β are constants. A is the technological parameter, α is the elasticity of output with respect to capital, and β is the elasticity of output with respect to labour.
- **Properties**
- Homogeneous of degree $(\alpha + \beta)$
- The returns to scale is immediately revealed by the sum of the two parameters α and β
 - **Constant Returns to Scale:** $(\alpha + \beta) = 1$
 - **Increasing Returns to Scale:** $(\alpha + \beta) > 1$
 - **Decreasing Returns to Scale:** $(\alpha + \beta) < 1$
- Isoquants are negatively sloped and convex to the origin
- $MRTS_{LK}$ is a function of input ratio
- Elasticity of substitution is equal to 1