Operation Research-II

Question :– On an average 96 patients per 24 hours day require the service of an emergency clinic. Also on the average a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs 100 per patient treated to obtain an average servicing time of 10 minutes and that each minute of decrease in the average time would cost Rs. 10 per patient treated. How would have to be budgeted by the clinic to decrease the average size of the queue from 4/3 patients to 1/2 a patient?

Sol:– $\lambda = \frac{96}{24 \times 60} = \frac{1}{15}$ /min

$\mu = \frac{1}{10}$ /min

$\rho = \frac{\lambda}{\mu} = \frac{10}{15} = 2/3$

$E(m) = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho} = \frac{4}{9(1-2/3)} = \frac{4}{3(1)}$

$= 4/3$

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Thus in order to decrease the queue size from $4/3$ to $1/2$, we have to determine the new value of μ

$\therefore$ Now, $E(m) = \dfrac{\lambda^2}{\mu(\mu-\lambda)}$

$$\Rightarrow \frac{1}{2} = \frac{(1/15)^2}{\mu\left(\mu - \frac{1}{15}\right)}$$

$$\Rightarrow \mu^2 - \frac{1}{15}\mu = 2/225$$

$$\Rightarrow \mu^2 - \frac{1}{15}\mu - 2/225 = 0$$

$$\Rightarrow 225\mu^2 - 15\mu - 2 = 0$$

$$\Rightarrow \mu = \frac{15 \pm \sqrt{225 - 4(-2)225}}{450}$$

$$= \frac{15 \pm \sqrt{225 + 1800}}{450}$$

$$= \frac{15 \pm \sqrt{2025}}{450}$$

$$= \frac{15 \pm 45}{450}$$

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$$= \frac{60}{450}, \frac{-30}{250}$$

$$= \frac{2}{15}$$

Avg time $= \frac{1}{\mu} = \frac{15}{2} = 7.5$ minutes

decrease in the time required to attend a patient

$$= 10 - \frac{15}{2} = \frac{5}{2} \text{ min.}$$

$\therefore$ Budget required for each patient

$$= \left(100 + \frac{5}{2} \times 10\right)$$

$$= 125$$

Question :- Assume that trains are coming in a yard at the rate of 30 per day and suppose that the inter-arrival times follow exponential distribution. The service time for each train is assumed to be Exponential with an average of 36 minutes. If the yard can admit 9 trains at a time (there being 10 lines and one is reserved for

Shunting purposes), then:

(i) Calculate that the probability that the yard is empty

(ii) Find the average queue length

Sol:- $N = 10$

$$\lambda = \frac{30}{24 \times 60} = \frac{1}{48} / \text{min}$$

$$\mu = \frac{1}{36} / \text{min}$$

$$\rho = \frac{\lambda}{\mu} = \frac{36}{48} = 0.75$$

(i) $$P_0 (\text{yard is empty}) = \frac{1-\rho}{1-\rho^{N+1}}$$

$$= \frac{1-0.75}{1-(0.75)^{11}}$$

$$= 0.26$$

(ii) $$E(m) = \frac{\rho^2 [1 - N\rho^{N-1} + (N-1)\rho^N]}{(1-\rho)(1-\rho^{N+1})}$$

$$= 1.76 \text{ Ans}$$