$(M/M/1):(N/FIFO)$ Model

The steady state difference eqⁿˢ of the model $(M/M/1):(\infty/FIFO)$ holds as long as $n<N$. The steady state eqⁿ at $n=N$ is derived as follows:

$$P_N(t+\Delta t) = P_N(t)[1-\mu\Delta t] + P_{N-1}(t)(\lambda\Delta t)[1-\mu\Delta t] + O(\Delta t)$$

$$\Rightarrow P_N(t+\Delta t) - P_N(t) = -P_N(t)(\mu\Delta t) + P_{N-1}(t)(\lambda\Delta t)[1-\mu\Delta t] + O(\Delta t)$$

$$\Rightarrow \frac{P_N(t+\Delta t) - P_N(t)}{\Delta t} = -P_N(t)(\mu) + P_{N-1}(t)(\lambda)[1-\mu\Delta t] + \frac{O(\Delta t)}{\Delta t}$$

$$\Rightarrow \frac{d}{dt}P_n(t) = -\mu P_N(t) + \lambda P_{N-1}(t)$$

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$0 = -\mu P_N + \lambda P_{N-1}$.

For steady state

Thus, the complete set of steady state eqⁿs of this model is given by:

$$\mu P_1 = \lambda P_0$$
$$\mu P_{n+1} = (\lambda+\mu) P_n - \lambda P_{n-1} \; ; \; 1 \le n \le N-1$$

$$\& \; \mu P_N = \lambda P_{N-1}$$

Thus, using iterative ~~prode~~ procedure, we have:

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 \; ; \quad n \le N-1$$

Thus for this model,

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 \; ; \quad n \le N$$

To calculate P_0, we make use of the condition, $\sum_{n=0}^{n} P_n = 1$

$$\Rightarrow 1 = P_0 \sum_{n=0}^{N} \rho^n = \begin{cases} P_0 \dfrac{1-\rho^{N+1}}{1-\rho} & , \rho \neq 1 \\ P_0 (N+1 & , \rho = 1 \end{cases}$$

Thus, $P_0 = \begin{cases} \dfrac{1-\rho}{1-\rho^{N+1}} & , \rho \neq 1 \\ \\ \dfrac{1}{N+1} & , \rho = 1 \end{cases}$

Hence $P_n = \begin{cases} \dfrac{(1-\rho)}{1-\rho^{N+1}} \rho^n & , \rho \neq 1 \\ \\ \dfrac{\rho^n}{N+1} & , \rho = 1 \end{cases}$

$0 \leq n \leq N$.