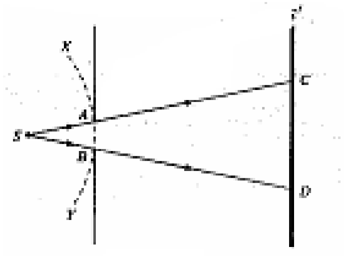
<u>**Diffraction of Light :**</u> The phenomenon of bending of light waves around edges of small obstacles and hence it's spreading into the geometrical shadow of the obstacle is called **diffraction**. The diffraction effects were first observed by **Grimaldi in 1665.** <u>**The effects can be observed only when the size of the obstacle is very small and comparable to the wavelength of light.**</u>



This phenomenon shows that the rectilinear propagation of light (light traveling along a straight) is only approximate i.e. light bends at the corners of small obstacles and enters the regions of geometrical shadows.

Light from the source S is made to fall on a slit AB whose width is very small. The region CD on the screen is found to contain unequally spaced alternate bright and dark fringes with light bending into the region above C and below D. This is due to diffraction effects. Fresnel explained this phenomenon by applying the Huygens 'Principle along with the principle of interference.

Diffraction phenomenon is classified into two types

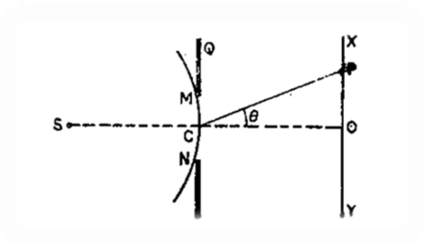
<u>**Fresnel's diffraction:**</u>	<u>**Fraunhofer's diffraction**</u>
1. The source of light and the screen on which the diffraction pattern is observed are at **finite distance** from the obstacle or aperture.	1. The source of light and the screen on which the diffraction pattern is observed are at **infinite distance** from the obstacle or aperture.
2. The incident wavefront and the diffracted wavefronts are spherical or cylindrical.	2. The incident wavefront and the diffracted wave fronts are plane wave fronts.
3. The incident beam is a divergent beam whereas the diffracted beam is a convergent beam.	3. The incident beam is a parallel beam and the diffracted beam is also parallel beam.
4. No changes in the wavefront are made by using either lenses or mirrors.	4. The incident rays from a source are made parallel using a convex lens and the diffracted rays are brought to focus on a screen using another convex lens (converging lenses).
5. The centre of the diffraction pattern is either bright or dark. The pattern is the image of the obstacle or aperture.	5. The centre of the diffraction pattern is always bright. The pattern is the image of the source itself.

Examples of diffraction – (1) The luminous border that surrounds the profile of a mountain just before sun rises behind it, (2) the light streaks that one sees while looking at a strong source of light with half shut eyes and (3) the coloured spectra one sees while viewing a distant source of light through a fine piece of cloth.

Fresnel's assumptions

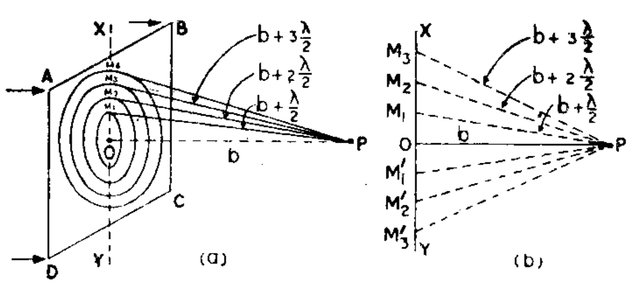
Fresnel in 1815, combined the Huygens principle of wavelet and the principle of interference to explain the bending of light around obstacles and also the rectilinear propagation of light.

1. According to Huygens' principle, each point of a wavefront (wavefront is a locus of points in a medium that are vibrating in same phase) is a source of secondary disturbance and wavelets coming from these points spread out in all directions with the speed of light. The envelope of these waves constitute the next wavelet.
2. According to Fresnel, a wavefront can be divided into a large number of strips or zones called Fresnel zones of small area. The resultant effect at any point will depend on the combined effect of all the secondary waves coming from various zones.
3. The effect at a point due to any particular zone depends on distance of the point from the zone.
4. The effect will also depend on the obliquity (inclination) of the point with reference to the zone under consideration.



Division of wavefront into Fresnel's half period zones – Expression for resultant displacement/amplitude – Rectilinear propagation of light

ABCD is a plane wave front of monochromatic light of wavelength λ. The diagram shows the plane wavefront as perpendicular to plane of the paper. Consider a point P at a distance b from the wave front at which amplitude due to the wave is to be found.



To find the resultant amplitude at P due to entire wavefront, Fresnel assumed the wavefront to be divided into a number of concentric half period zones called

Fresnel's half period zones:

With P as centre and with $M_1P = \left(b + \frac{\lambda}{2}\right), M_2P = \left(b + \frac{2\lambda}{2}\right), \ldots\ldots$ as radii, a series of concentric spheres are drawn on the wavefront. These spheres intersect the wavefront in concentric circles. These circles or zones are of radii OM_1, OM_2 ,….. on the wavefront with O as centre.

The secondary waves from any two consecutive zones reach the point P with a path difference of $\frac{\lambda}{2}$ or a time period of $\frac{T}{2}$. Hence these zones are called half period zones. The area of the circle OM_1 is called first half period zone. The area between the circles of OM_2 and OM_1 is called second half period zone and so on. The area between the nth and (n – 1)th circle is called the nth half period zone.

To find the radius of a half period zone :

In the diagram, from the right angled triangle OM_1P,

$$OM_1 = \sqrt{(M_1P)^2 - (OP)^2} = \sqrt{\left(b + \frac{\lambda}{2}\right)^2 - b^2}$$

$OM_1 = \sqrt{\left(b^2 + 2b\frac{\lambda}{2} + \frac{\lambda^2}{4}\right) - b^2}$ or $OM_1 = \sqrt{b\lambda}$ (neglecting $\frac{\lambda^2}{4}$ as $b >> \lambda$)

$\boldsymbol{OM_1 = \sqrt{b\lambda}}$ is the radius of first half period zone.

The radius of the second half period zone is

$OM_2 = \sqrt{(M_2P)^2 - (OP)^2} = \sqrt{\left(b + \frac{2\lambda}{2}\right)^2 - b^2}$ Thus $\boldsymbol{OM_2 = \sqrt{2b\lambda}}$

Similarly the radius of the nth half period zone is $OM_n = \sqrt{\left(b + \frac{n\lambda}{2}\right)^2 - b^2}$

or $\boldsymbol{OM_n = \sqrt{nb\lambda}}$

Thus the radii of 1st , 2nd , …… half period zones are $\sqrt{b\lambda}, \sqrt{2b\lambda}, \ldots\ldots\sqrt{nb\lambda}$.

Therefore, the radii of the zones are proportional to the square root of natural numbers.

To find the area of half period zones :

The area of first half period zone is

$= \pi\,(OM_1)^2 = \pi[(M_1P)^2 - (OP)^2]$ (As area = πr^2)

$= \pi\left[\left(b + \frac{\lambda}{2}\right)^2 - b^2\right] = \pi\, b\lambda$

The area of 2th half period zone $= \pi[(OM_2)^2 - (OM_1)^2]$

$= \pi[2b\lambda - b\lambda] = \pi\, b\lambda$

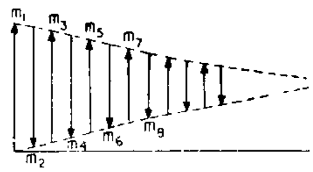
The area of nth half period zone $= \pi[(OM_n)^2 - (OM_1)^2]$

$= \pi[nb\lambda - b\lambda] = \boldsymbol{\pi\, b\lambda}$

Thus the area of each half period zone is same and is equal to λ .

Also, the area of any zone is directly proportional to the wavelength (λ) of light and the distance of the point from the wavefront (b).

<u>To find amplitude due to the wavefront :</u>



The amplitude of the waves at P due to an individual zone is

1. Directly proportional to the area of the zone
2. inversely proportional to the distance of the point P from the given zone.
3. the obliquity factor (1+ cos θ) where θ is the angle between normal to the zone and the line *joining* the zone to the point P. The effect at P decreases as obliquity increases.

The path difference between any two consecutive half period zones is $\frac{\lambda}{2}$. Hence the waves from two consecutive zones will reach P in opposite phase. If m_1, m_2, m_3 ,….. are the amplitudes at P due to 1st, 2nd, 3rd, ….. half period zones, the resultant amplitude at P due to entire wavefront is

$A = m_1 - m_2 + m_3 - m_4 + \cdots . + m_n$ if n is odd

and $A = m_1 - m_2 + m_3 - m_4 + \cdots . - m_n$ if n is even.

As the obliquity increases amplitudes decreases, ie. m_2 is less than m_1, m_3 is less than m_2 etc…

Thus on the average $m_2 = \frac{m_1 + m_3}{2}$ ….(1) Similarly $m_4 = \frac{m_3 + m_5}{2}$. ……..(2)

The equation $A = m_1 - m_2 + m_3 - m_4 + \cdots$. can be written as

$A = \frac{m_1}{2} + \left(\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right) + \left(\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right) + \cdots$ ……..(3)

Substituting equations (1) and (2) in (3) we get

$A = \frac{m_1}{2} + \frac{m_n}{2}$ if n is odd ….(5) (The terms in the bracket cancel)

$A = \frac{m_1}{2} + \frac{m_{n-1}}{2} - m_n$ if n is even. ….(6).

As the amplitudes are of diminishing order, for large n, m_n and m_{n-1} tend to zero. Thus $\boldsymbol{A = \frac{m_1}{2}}$.

The amplitude of the wave at any point P, in front of a large plane wavefront is equal to half the amplitude due to the first half period zone.

As the intensity is proportional to square of the amplitude, ($I \propto A^2$ the intensity at P is proportional to $\frac{{m_1}^2}{4}$ $\left(I \propto \frac{{m_1}^2}{4}\right)$. Thus the intensity at point P is one fourth of the intensity due to the first half period zone.

Explanation of rectilinear propagation of light

The intensity at point in front of a wave front is proportional to $\frac{{m_1}^2}{4}$ where m_1 is the amplitude of the first half period zone. Thus the intensity at point P is one fourth of the intensity due to the first half period zone.

Thus only half the area of the first half period zone is effective in producing the illumination at the point P. A small obstacle of the size of half the size of half the area of first half period zone placed at O will block the effect of whole wavefront and the intensity at P due to rest of the wavefront is zero.

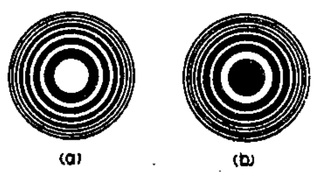
While dealing with the rectilinear propagation of light, the size of the obstacle used is far greater than the area of the first half period zone and hence the bending effect of light round the corners of the obstacle is diffraction effects cannot be noticed.

Thus if the size of the obstacle placed in the path of light is very small and comparable to wavelength of light, then it is possible to observe illumination in the region of the geometrical shadow also. **Thus, rectilinear propagation of light is only approximately true.**

Zone plate

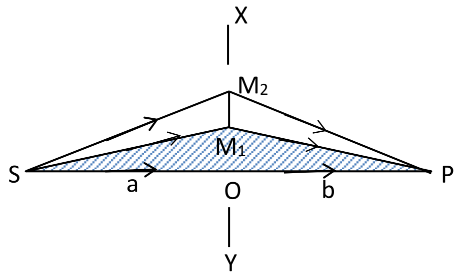
A zone plate is a specially constructed screen such that light is obstructed from every alternate zone.The correctness of Fresnel's method in dividing a wavefront into half period zones can be verified with the help of zone plate.

A zone plate is constructed by drawing concentric circles on a white paper such that radii are proportional to the square root of the natural numbers. The odd numbered zones (i.e. 1st, 3rd, 5th …) are covered with black ink and a reduced photograph is taken.



The negative of the photograph appears is as shown in Fig. (a). The negative shows odd zones are transparent to incident light and even zones will cut off light. This is a **positive zone plate**. If odd zones are opaque and the even zones are transparent then it is a **negative zone plate**. Fig. (b)

Theory



Let S be a point source of light of wavelength λ placed at a distance a from centre O of the zone plate. Let P be the point on a screen placed at distance b at which intensity of diffracted light bright.

Let r_1, r_2,r_3…….r_n be the radii of the 1st, 2nd, 3rd ……nth half period zones respectively. The position of the screen is such, that from one zone to the next there is an increasing path difference of $\frac{\lambda}{2}$.

Thus, from the diagram $SO + OP = a + b$

$$SM_1 + M_1P = a + b + \frac{\lambda}{2} \qquad ....(1)$$

Similarly $SM_2 + M_2P = a + b + \frac{2\lambda}{2}$ and so on

From the triangle SM_1O $\quad SM_1 = (SO^2 + OM_1^2)^{1/2} = (a^2 + r_1^2)^{1/2}$

Similarly from the triangle PM_1O $\quad M_1P = (OP^2 + OM_1^2)^{1/2} = (b^2 + r_1^2)^{1/2}$

Substituting the values of SM_1 and M_1P in equation (1), we get

$$(a^2 + r_1^2)^{1/2} + (b^2 + r_1^2)^{1/2} = a + b + \frac{\lambda}{2}$$

or $\quad a\left(1 + \frac{r_1^2}{a^2}\right)^{1/2} + b\left(1 + \frac{r_1^2}{b^2}\right)^{1/2} = a + b + \frac{\lambda}{2}$

Expanding and simplifying the above equation, we get

$$a\left(1 + \frac{r_1^2}{2a^2}\right) + b\left(1 + \frac{r_1^2}{2b^2}\right) = a + b + \frac{\lambda}{2}$$

$$a + \frac{r_1^2}{2a} + b + \frac{r_1^2}{2b^2} = a + b + \frac{\lambda}{2}$$

or $\quad \frac{r_1^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{\lambda}{2} \qquad$ or $\quad r_1^2\left(\frac{1}{a} + \frac{1}{b}\right) = \lambda$

Thus for the radius of the nth zone the above relation can be written as

$r_n^2\left(\frac{1}{a} + \frac{1}{b}\right) = n\,\lambda$ …..(2) $\quad$ or $\quad r_n^2 = \frac{ab}{a+b} n\,\lambda \quad$ or $r_n = \sqrt{\frac{ab\lambda}{a+b}}\,\sqrt{n}$

Thus the radii of the half period zones are proportional to the square root of the natural numbers.

From equation (2) can written as $\left(\frac{\boldsymbol{1}}{\boldsymbol{a}}+\frac{\boldsymbol{1}}{\boldsymbol{b}}\right)=\frac{\boldsymbol{n\,\lambda}}{\boldsymbol{r_n^2}}$ …..(3)

This equation is similar to the lens formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ …..(4)

Comparing equations (3) and (4) $\frac{\boldsymbol{1}}{\boldsymbol{f}}=\frac{\boldsymbol{n\,\lambda}}{\boldsymbol{r_n^2}}$ **or** $\boldsymbol{f}=\frac{\boldsymbol{r_n^2}}{\boldsymbol{n\,\lambda}}$

f is the focal length of zone plate and acts as a convex lens of multiple foci.

The path difference between any successive transparent zones is λ and the phase difference is 2π. Waves from successive zones reach P in phase.

Focussing action of Zone plate

The amplitude at P depends on (a) area of the zone, (b) distance of the zone from P and (c) obliquity factor.

The area of nth zone = $\pi r_n^2 - \pi\, r_{n-1}^2$

As $r_n^2 = \frac{ab}{a+b} n\,\lambda,$ the area of the nth zone $=\frac{ab}{a+b}n\,\lambda - \pi\,\frac{ab}{a+b}(n-1)\,\lambda = \pi\,\frac{ab\lambda}{a+b}$

Area is independent of n. Area of all zones are same. But the distance of the zone from P and obliquity factor increases as n increases.

The resultant amplitude at P is

A = $m_1 + m_3 + m_5 + \ldots$.for positive zone plate

A = - $(m_2 + m_4 + m_6 + \ldots\ldots)$ for negative zone plate.

This is much greater than A = $\frac{m_1}{2}$ which is due to all zones.

As the intensity from the zone plate is very high, the zone plate is said to have focussing action

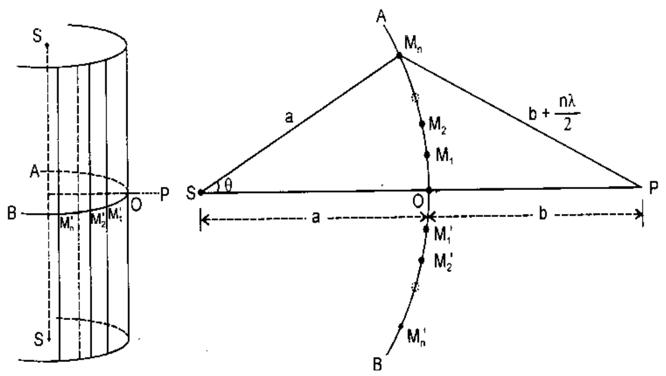
Differences between Zone plate and Convex lens

Zone plate	Convex lens
Focal length of a zone plate is $\frac{\boldsymbol{1}}{\boldsymbol{f}}=\frac{\boldsymbol{n\,\lambda}}{\boldsymbol{r_n^2}}$	Focal length of lens is $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$
f depends on λ and show chromatic aberration. Forms real image.	f depends on λ and show chromatic aberration. Forms real image.
It has multiple foci. If (2p -1) is the number of half period elements in each zone $\boldsymbol{f_p}=\frac{\boldsymbol{r_n^2}}{(\boldsymbol{2p-1})\boldsymbol{n\,\lambda}}$	It has single focus. $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Zone plate	Convex lens
All the waves reaching the image point through consecutive transparent zones have a path difference of λ .	All waves reaching the image point have same optical path.
fviolet > fred	fviolet < fred
Intensity of image is less	Intensity of image is greater.

Theory of Cylindrical half period strips

S is a narrow rectangular slit or a linear source of light of wavelength λ. AB is the cylindrical wavefront. P is a point on the axis of the wavefront at which resultant intensity is to be found.



<u>To find the resultant amplitude/intensity at P due to the wavefront</u>

If m_1, m_2, m_3 ……are the amplitudes at P due to 1st, 2nd, 3rd, …… half period strips on either side of O, the resultant amplitude due to one half of the wavefront is $A = m_1 - m_2 + m_3 - m_4 + \ldots$

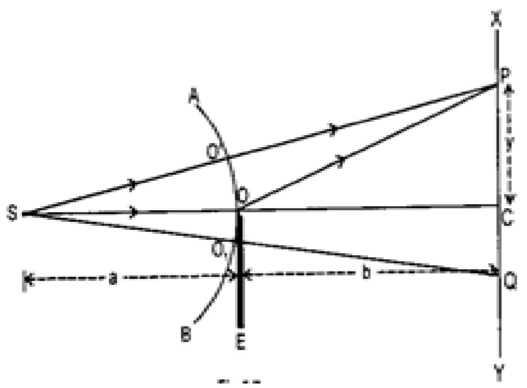
$$A = \frac{m_1}{2} + \left(\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right) + \left(\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right) + \ldots$$

$$A = \frac{m_1}{2} \quad \left(\text{since } \frac{m_1 + m_3}{2} = m_2\right)$$

Hence the resultant amplitude due to entire wavefront is $\boldsymbol{A = \frac{m_1}{2} + \frac{m_1}{2} = m_1}$

Diffraction at a straight edge

<u>Theory</u> - S is a narrow rectangular slit illuminated light of wavelength λ. OE is the straight edge (opaque object covering half the slit in vertical plane), AB is the cylindrical wavefront, XY is the screen. CX is the region of diffraction fringes and CY has illuminationof decreasing intensity.



The intensity at P at distance y from C on the screen is due to upper part of wavefront AB above O′ and that due to exposed portion OO′ of the wavefront.

The amplitude at P due to upper half above O′ is = $\frac{m_1}{2}$.

If portion OO′ contains one half period strip,

the total amplitude at P = $\frac{m_1}{2} + m_1 = \frac{3m_1}{2}$ (maximum amplitude)

If OO′ contains two half period strips,

then the total amplitude at P = $\frac{m_1}{2} + m_1 - m_2 = \frac{3m_1}{2} - m_2$ (minimum amplitude)

When region OO′ has odd half period strips, the amplitude is maximum and for even half period strips it is minimum.

Above C, on the screen a diffraction pattern of alternate maxima and minima are observed. As distance on the screen increases, the intensity becomes uniform.

<u>Conditions for maxima and minima of diffraction pattern</u>

The path difference between waves from O and O′ reaching P is $\delta = PO - PO'$

If $\boldsymbol{\delta}$ is odd multiples of $\frac{\lambda}{2}$, amplitude at P is maximum.

i.e. $\boldsymbol{\delta = PO - PO' = (2n+1)\frac{\lambda}{2}}$ .....(1)

If $\boldsymbol{\delta}$ is even multiples of $\frac{\lambda}{2}$, amplitude at P is minimum.

i.e. $\boldsymbol{\delta = PO - PO' = 2n\frac{\lambda}{2} = n\lambda}$ ……(2)

where n = 0, 1, 2, 3, ……

From the diagram, $PO = \sqrt{(OC)^2 + (CP)^2}$

$$= \sqrt{b^2 + y^2} = b\left[1 + \frac{y^2}{b^2}\right]^{1/2} = b\left[1 + \frac{y^2}{2b^2}\right]$$

Thus $PO = b + \frac{y^2}{2b}$

and $PO' = SP - SO' = \sqrt{(SC)^2 + (CP)^2} - SO'$

$$= \sqrt{(a+b)^2 + y^2} - a$$

$$= (a+b)\left[1 + \frac{y^2}{(a+b)^2}\right]^{1/2} - a$$

Thus $PO' = (a+b)\left[1 + \frac{y^2}{2(a+b)^2}\right] - a = b + \frac{y^2}{2(a+b)}$

Hence $PO - PO' = b + \frac{y^2}{2b} - b - \frac{y^2}{2(a+b)}$

$PO - PO' = \frac{ay^2 + by^2 - by^2}{2b(a+b)} = \frac{ay^2}{2b(a+b)}$ …..(3)

Comparing equation (3) with (1), the condition for maximum is

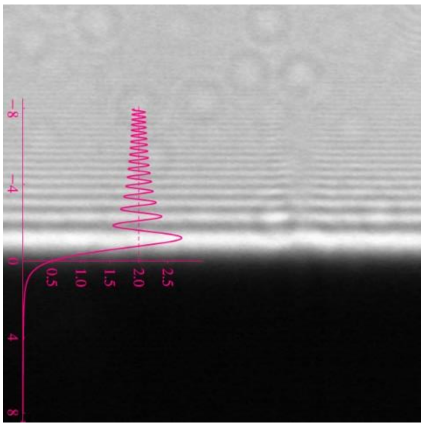
$$\frac{ay^2}{2b(a+b)} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad y^2 = \frac{2b(a+b)\,(2n+1)\lambda}{2a}$$

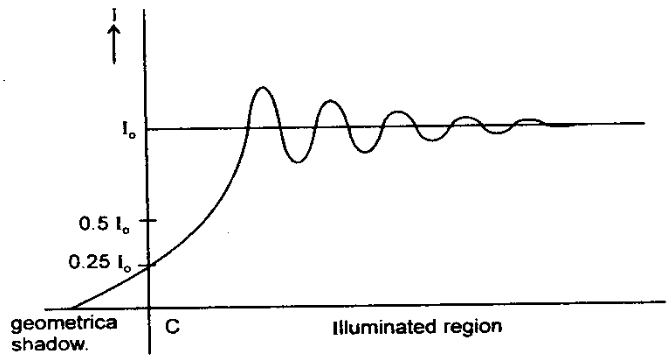
or $\boldsymbol{y_n} = \sqrt{\frac{\boldsymbol{b(a+b)\,(2n+1)\lambda}}{\boldsymbol{a}}}$. This is distance of n^th maximum from the centre C.

Comparing equation (3) with (2), the condition for minimum is $\frac{ay^2}{2b(a+b)} = n\lambda$

or $\boldsymbol{y_n} = \sqrt{\frac{\boldsymbol{2b(a+b)n\lambda}}{\boldsymbol{a}}}$. This is distance of n^th minimum from the centre C.

The diagram shows the diffraction pattern due to straight edge.





The graph shows the intensity distribution due to diffraction at a straight edge. The intensity on the screen XY is due to upper part of wavefront AB only as the lower half is blocked.

The resultant amplitude at C is $\frac{m_1}{2}$ and the intensity at C is $\frac{{m_1}^2}{4}$. It is the one fourth of the intensity compared to intensity when entire wavefront is exposed.

<u>Intensity within the geometrical shadow</u>

For a point Q on the screen in the shadow region, O_1 is the pole of wavefront AB. Light from region below O_1 is cut off. Also part of upper region OO_1 is cut off. If the region above O_1 till O cuts off only first half period zone, then amplitude at Q is due to other zones given by $m_2 - m_3 + m_4 - m_5 \ldots\ldots = \frac{m_2}{2}$

If it cuts off two zones, then amplitude $= \frac{m_3}{2}$ and so on.

Thus intensity decreases rapidly initially and then slowly as we move further into geometrical shadow.

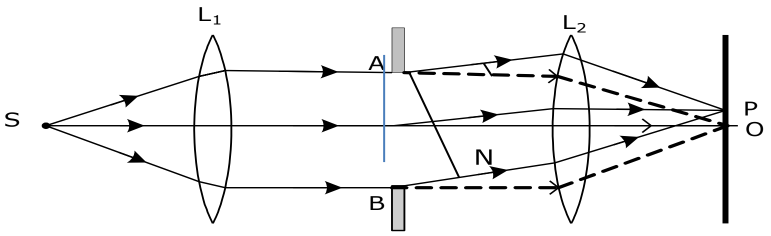
Fraunhofer diffraction

The phenomenon of bending of light waves around edges of small obstacles and hence it's spreading into the geometrical shadow of the obstacle is called **diffraction**. In case of Fraunhofer diffraction -

- The source of light and the screen are at **infinite distance** from the obstacle or aperture.
- The incident wavefront and the diffracted wave fronts are plane.
- The incident beam and diffracted beams are parallel. Convex lenses are used to make the wavefront parallel.

Fraunhofer diffraction at a single slit

Consider a point source of light S placed at the principle focus of lens L_1. The parallel rays strike the single slit AB. Each Point on the slit AB act as sources of secondary disturbances and sends out secondary waves in all directions.



The diffracted rays after passing through lens L_2 are brought to focus on the screen. The diffraction pattern consists of central bright and alternate bright and dark bands of decreasing intensity.

The waves travelling from A and B reaching O are in phase. Thus the path difference between AO and BO is zero. The waves superpose constructively resulting in central maximum at O (Bright region)

A perpendicular is drawn from A to the line BP. BN is the path difference between the waves travelling from Aand B reaching P. It is given by ***BN = d sinθ***

(Since, from right angled triangle ABN, $sin\theta = \frac{BN}{AB}$ or $BN = ABsin\theta$ and AB = d.

As Phase difference = $\frac{2\pi}{\lambda}$ × path difference

Thus Phase difference = $\frac{2\pi}{\lambda}$ × d sinθ

If number of equal parts to which the wavefront AB is divided = n,

Phase difference between any two consecutive waves from these parts is

= $\frac{1}{n}$ × total phase = $\frac{1}{n} \times \frac{2\pi}{\lambda}$ × d sinθ = ϕ (say)

From the method of vector addition, the resultant amplitude $R = a\,\frac{sin\frac{n\phi}{2}}{sin\frac{\phi}{2}}$

where a is the amplitude of the wave from each part

$$R = a\,\frac{\sin\frac{n}{2}\times\frac{1}{n}\times\frac{2\pi}{\lambda}\times d\sin\theta}{\sin\frac{1}{2}\times\frac{1}{n}\times\frac{2\pi}{\lambda}\times d\sin\theta} = a\,\frac{\sin\alpha}{\sin\frac{\alpha}{n}}$$

where $\boldsymbol{\alpha} = \dfrac{\boldsymbol{\pi d\sin\theta}}{\boldsymbol{\lambda}}$ Since $\frac{\alpha}{n}$ is very small, $\sin\frac{\alpha}{n} = \frac{\alpha}{n}$

Thus $R = a\,\dfrac{\sin\alpha}{\frac{\alpha}{n}} = na\,\dfrac{\sin\alpha}{\alpha} = A\,\dfrac{\sin\alpha}{\alpha}$ or the resultant amplitude is $R = A\,\dfrac{\sin\alpha}{\alpha}$

(when n → ∞, $a \to 0$ but $\mathrm{A} = na$ remains finite).

As intensity is directly proportional to square of amplitude

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Condition for principal maximum

$$\mathrm{R} = A\,\frac{\sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \ \dots\right]$$

or $\quad \mathrm{R} = A\left[1 - \dfrac{\alpha^2}{3!} + \dfrac{\alpha^4}{5!} - \dfrac{\alpha^6}{7!} + \ \dots\right]$

If the negative terms vanish, the value of R is maximum,

i.e. $\alpha = 0$, $\therefore \alpha = \dfrac{\pi d\sin\theta}{\lambda} = 0$ or $\sin\theta = 0.$ Thus $R = A$ (max. amplitude and max. Intensity). This corresponds to principal maximum.

Condition for minimum intensity

The intensity will be minimum when $\sin\alpha = 0.$ The values of α which satisfy this equation are $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm \ \dots\dots. = \pm m\pi$

or $\alpha = \dfrac{\pi d\sin\theta}{\lambda} = \pm m\pi$ or $\boldsymbol{d\sin\theta = \pm m\lambda}$ where *m = 1, 2, 3, …..*

This condition corresponds to minimum intensity.

Condition for secondary maxima

As $I = A^2\left(\dfrac{\sin\alpha}{\alpha}\right)^2$ Differentiating this equation w.r.t α and equating to zero,

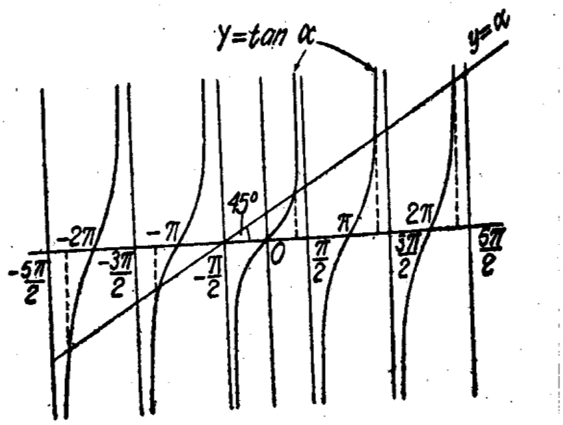
$\dfrac{dI}{d\alpha} = \dfrac{d}{d\alpha}\left[A^2\left(\dfrac{\sin\alpha}{\alpha}\right)^2\right] = 0$ we get $A^2\,\dfrac{2\sin\alpha}{\alpha}\left[\dfrac{\alpha\cos\alpha - \sin\alpha}{\alpha^3}\right] = 0$

Thus either $\sin\alpha = 0$ or $\alpha\cos\alpha - \sin\alpha = 0$

$\sin\alpha = 0$ gives condition for minimum. Hence positons of maxima are given by roots of the equation $\boldsymbol{\alpha\cos\alpha - \sin\alpha = 0}$ **or** $\boldsymbol{\alpha = \tan\alpha}$

The values of α satisfying $\alpha = tan\alpha$ are obtained graphically by plotting the curves, $y = \alpha$ and $y = \tan\alpha$ on the same graph.

The points of intersection of the two curves gives the values of α which satisfy $y = \tan\alpha$



The points of intersection are

$\alpha = 0, \ \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, ..$

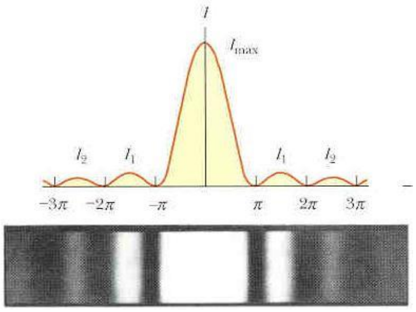
or $\alpha = 0, \ \pm 1.430\pi, \pm 2.462\pi, \ldots\ldots$

Here $\alpha = 0$, gives principal maximum. $(I_0 = A^2)$

Putting the values of α in the eqn. $I = A^2\left(\frac{sin\alpha}{\alpha}\right)^2$ $\quad I_1 = A^2\left(\frac{sin3\pi/2}{3\pi/2}\right)^2 = \frac{A^2}{22}$

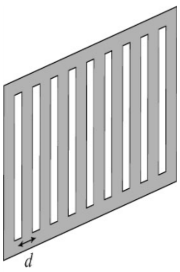
(first subsidiary maximum or first secondary maximum)

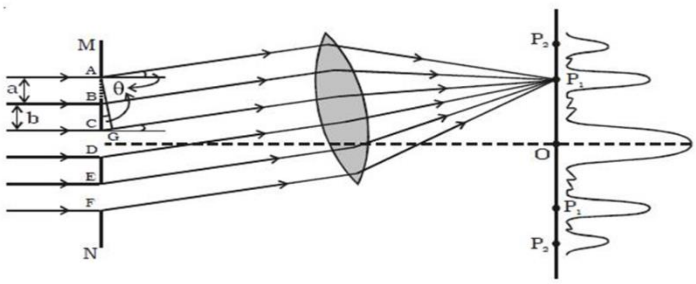
$I_2 = A^2\left(\frac{sin5\pi/2}{5\pi/2}\right)^2 = \frac{A^2}{62}$ (second subsidiary maximum or secondary maximum) and so on.



The graph indicates the intensity distribution in case of diffraction due to single slit.

Plane diffraction grating





An arrangement consisting of a large number of parallel slits of same width and separated by equal opaque spaces is called diffraction grating.

If a – width of each slit and b – width of each opaque space, then (a + b) is called grating element.

Theory of diffraction grating (normal incidence)

Consider parallel beam of light striking the transmission diffraction grating MN. The waves from different slits superpose and produce diffraction pattern on the screen. The pattern consists of a number of principal maxima with minima and secondary maxima in between. The incident beam travelling in the same direction will be brought to focus at O which corresponds to central maximum.

To find the intensity at P_1 - Fraunhofer diffraction at a single slit is applied.

The wavelet travelling from all the points in a slit along the direction θ are equivalent to a single wave of amplitude $R = A\frac{sin\alpha}{\alpha}$ where $\alpha = \frac{\pi d sin\theta}{\lambda}$ t

If there are N slits, there are N waves each from middle of the slits.

The path difference between any two consecutive slits is

$$\delta = CG = AC\ sin\theta = (a+b)sin\theta$$

[From diagram above, consider the triangle ACG, where CG is the path difference and $sin\theta = CG/AC$ where AC = (a + b)

The phase difference = $\frac{2\pi}{\lambda} \times (a+b)\sin\theta$

This is a constant and let it be equal to 2β

$$2\beta = \frac{2\pi}{\lambda} \times (a+b)\sin\theta \quad \text{or} \quad \beta = \frac{\pi(a+b)\sin\theta}{\lambda}$$

By the method of vector addition of amplitudes, the resultant amplitude in the direction of θ is $R = A\frac{sin\alpha}{\alpha}\ \frac{sinN\beta}{sin\beta}$

[By vector addition $R = a\frac{sin\frac{n\phi}{2}}{sin\frac{\phi}{2}}$ and here $a = A\frac{sin\alpha}{\alpha}$, $n = N\ and\ \phi = 2\beta$

The resultant intensity $I = R^2 = \left(A\frac{sin\alpha}{\alpha}\right)^2\left(\frac{sinN\beta}{sin\beta}\right)^2$

The factor $\left(A\frac{sin\alpha}{\alpha}\right)^2$ gives distribution of intensity due to single slit and the factor $\left(\frac{sinN\beta}{sin\beta}\right)^2$ gives distribution of intensity as combined effects of all the slits.

Condition for principal maxima

The intensity would be maximum when $sin\beta = 0$,

or $\beta = \pm n\pi$ where n = 0, 1, 2, …

At the same time $sinN\beta = 0$, so that the factor $sinN\beta/sin\beta$ becomes indeterminate.

It is evaluated as follows

$$\lim_{\beta\to\pm n\pi}\frac{sinN\beta}{sin\beta} = \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(sinN\beta)}{\frac{d}{d\beta}(sin\beta)} = \lim_{\beta\to\pm n\pi}\frac{NcosN\beta}{cos\beta} = \pm N$$

Hence $\lim_{\beta \to \pm n\pi} \left(\frac{sinN\beta}{sin\beta}\right)^2 = N^2$

The resultant intensity is $\mathrm{I} = R^2 = \left(A\frac{sin\alpha}{\alpha}\right)^2 N^2$

The maxima are referred to as **principal maxima**.

The maxima are obtained for $\beta = \ \pm n\pi$ or $\beta = \frac{\pi(\mathrm{a}+\mathrm{b})\sin\theta}{\lambda} = \pm n\pi$

or $\boldsymbol{(a\ +\ b)\ sin\theta =\ \pm n\lambda}$ where *n = 0, 1, 2, 3,…….*

n = 0 → central maximum

n = 1, 2, 3,…. → first, second, third,… … principal maxima.

<u>Condition for minima</u>

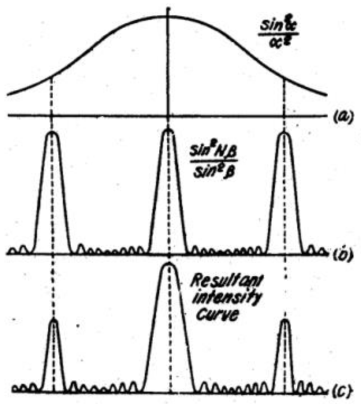
A number of minima occur, when $sinN\beta = 0$ but $n\beta \neq 0$.

Thus $sinN\beta = 0$ implies $N\beta = \pm m\pi$

$N\beta = N\frac{\pi(\mathrm{a}+\mathrm{b})\sin\theta}{\lambda} = \pm m\pi$

or $\mathrm{N(a\ +\ b)\ sin\theta} = \pm m\lambda$ where m has all integral values except *0. N, 2N,…….nN,* since for these values $sin\beta = 0$ corresponding to principal maxima. Thus *m = 1, 2, 3, ……(N – 1).*

<u>Condition for secondary maxima</u>



As there are (N – 1) minima between two adjacent principal maxima, there must be (N – 2) other maxima between two principal maxima. These are known as secondary maxima. As N becomes large, intensity of these maxima decreases relative to principal maxima and become negligible.

The graph shows the intensity distribution curve under different conditions as shown.

<u>Width of principal maxima in the diffraction pattern</u>

The condition for the nth maxima in the diffraction pattern due to grating is given by

$(a\ +\ b)\ sin\theta_n =\ n\lambda$ …..(1)

or $N(a\ +\ b)\ sin\theta_n =\ Nn\lambda,$ ……(2) N – number of slits

Considering minima on either side of principal maxima,

then its direction is $(\theta_n \pm d\theta_n)$ with $d\theta_n$ → angular half width of the nth maximum.

For the first minima of the n[th] principal maxima $m = Nn \pm 1$

$N(a + b)\, sin(\theta_n \pm d\theta_n) = (Nn \pm 1)\lambda$ ……..(3)

$N(a + b)[sin\theta_n \cos d\theta_n \pm \cos\theta_n\, sind\theta_n] = (Nn \pm 1)\lambda$

Since $d\theta_n$ is very small, cos $d\theta_n$ =1 and sin$d\theta_n$= $d\theta_n$

Thus $N(a + b)[sin\theta_n \pm \cos\theta_n\, d\theta_n] = (Nn \pm 1)\lambda$

$N(a + b)\, sin\theta_n \pm N(a + b)cos\theta_n\, d\theta_n = (Nn \pm 1)\lambda$ …..(4)

Using condition (2) in (4) $Nn\lambda \pm N(a + b)cos\theta_n\, d\theta_n = Nn\lambda \pm \lambda$

$N(a + b)cos\theta_n\, d\theta_n = \lambda$ or $d\theta_n = \frac{\lambda}{N(a+b)cos\theta_n}$

The angular width of n[th] principal maxima is given by

$$\boldsymbol{(\theta_n + d\theta_n) - (\theta_n - d\theta_n) = 2d\theta_n = \frac{2\lambda}{N(a+b)cos\theta_n}}$$

Maximum number of orders available with a grating

For the principal maxima $(a + b)\, sin\theta = n\lambda$

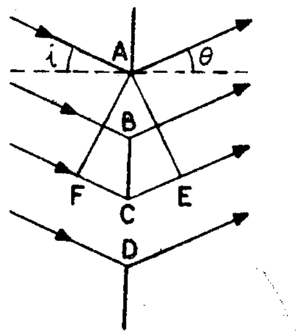
or $n = \frac{(a+b)\, sin\theta}{\lambda}$

The maximum angle of deflection is θ = 90⁰, the maximum order is $n_{max} = \frac{(a+b)}{\lambda}$

If the grating element is less than twice the λ, then (a + b) $< 2\lambda$ or $n_{max} < 2\lambda/\lambda <$ 2 . Thus only first order is possible.

Theory of diffraction grating (oblique incidence)

Consider parallel beam of light of wavelength λ incident on a grating at oblique incidence as shown in the diagram. AB is a slit of width 'a' and BC is the opaque region of width 'b'. The path difference between the waves from points A and C is $\delta = FC + CE$



From triangle AFC, $FC = (a + b)\, sin\, i$

From triangle, AEC, $CE = (a + b)\, sin\theta$

$\therefore \delta = FC + CE = (a + b)[sin\, i + sin\theta]$

For the n[th] principal maximum,

$(a + b)[sin\,\theta_n + sini] = n\lambda$ …..(1)

$(a + b)\left[2\, sin\frac{\theta_n + i}{2}\, cos\frac{\theta_n - i}{2}\right] = n\lambda$ or $sin\frac{\theta_n + i}{2} = \frac{n\lambda}{2(a+b)cos\frac{\theta_n - i}{2}}$

The total deviation of the diffracted beam $d = \theta_n + i$. For the deviation to be minimum, $\sin\frac{\theta_n+i}{2}$ should be minimum,

i.e. $cos\frac{\theta_n-i}{2}$ should be maximum. i.e. $\frac{\theta_n-i}{2} = 0$ or $\theta_n = i$.

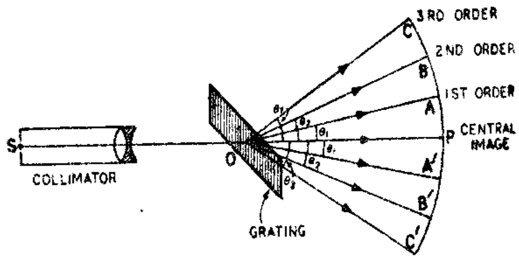
If D_m is angle of minimum deviation, $D_m = \theta_n + i$, As $\theta_n = i$,

Thus $i = \theta_n = \frac{D_m}{2}$ Thus the condition for principal maximum is

$$\boldsymbol{2(a+b)\left[sin\frac{D_m}{2}\right] = n\lambda} \text{ (from (2))}$$

Detrmination of wavelength of spectral line using grating

The condition for principal maximum in case of diffraction pattern due to grating is $(a + b)\ sin\theta = n\lambda$ .....(1)



(a + b) is the grating element. If N is the Number of lines on the grating per inch, then $(a + b) = \frac{2.54}{N}\ cm$. To find λ, the angle of diffraction θ is to be determined experimentally and the above equation is to be used.

Equation (1) also shows that for a particular wavelength λ, the angle of diffraction θ is different for principal maxima of different orders. Also for white light and for a particular order n, the light of different wavelengths will be diffracted in different directions. The longer the wavelength, greater is the angle of diffraction. So in each order there will be as many lines as there are wavelengths. Thus n = 0 corresponds to central maximum where all wavelengths coincide and a white colour is seen. Also n = 1 correspond to first order of all coloured lines corresponding to different wavelengths with violet being the innermost colour and red being the outermost colour. Similar if for other orders.

Dispersive power of grating

Dispersive power of a gating is its ability to split the white light into its constituent colours and show them distinctly.

$$Dispersive\ power\ (\omega) = \frac{change\ of\ angle\ of\ diffraction}{change\ in\ wavelength\ of\ light} = \frac{d\theta}{d\lambda}$$

For a grating $(a + b)\ sin\theta = n\lambda$

Differentiating w.r.t λ, $(a + b)\,cos\theta\frac{d\theta}{d\lambda} = n$

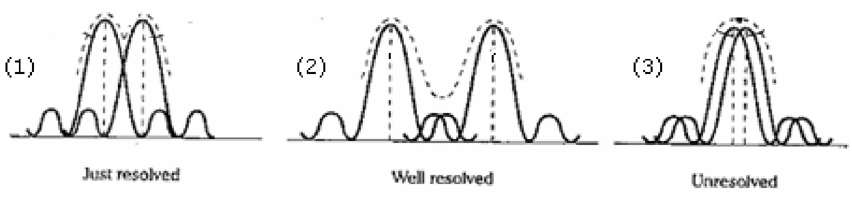
Thus $\boldsymbol{\omega} = \frac{\boldsymbol{d\theta}}{\boldsymbol{d\lambda}} = \frac{\boldsymbol{n}}{(\boldsymbol{a}+\boldsymbol{b})\,\boldsymbol{cos\theta}}$

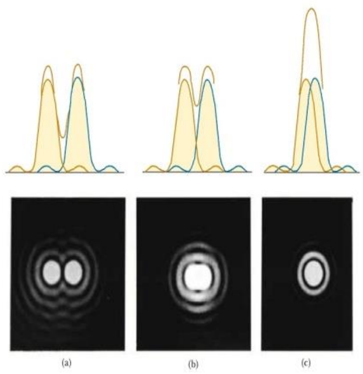
From the above equation it is clear that the dispersive power is directly proportional to the order n, inversely proportional to the grating element and inversely proportional to cosθ, i.e. larger the value of θ, smaller the value of cosθ, and higher is the dispersive power.

Resolving power

The ability of an optical instrument to show two close lying point objects as well separated point objects is called its **resolving power**. The resolution is limited by the diffraction patterns of the two close lying point objects which overlap as shown.

Rayleigh criterion for resolution

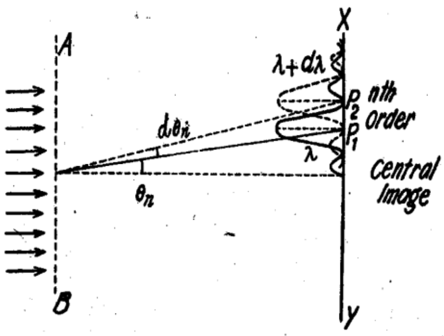




1. **Condition for just resolved** – Two close lying sources of light or point objects are said to be just resolved, if the central maximum of the diffraction pattern due to one source coincides with the first minimum of the diffraction pattern due to the second source. It also means that the distance between two central maxima due to two sources is **equal to** the distance between the central maximum and first minimum of any one of them.
2. **Condition for well resolved** – Two close lying sources of light or point objects are said to be well resolved, if the distance between two central maxima of the diffraction pattern due to two sources is **greater than** the distance between the central maximum and first minimum of any one of them.
3. **Condition for unresolved** – Two close lying sources of light or point objects are said to be unresolved, if the distance between two central maxima of the

diffraction pattern due to two sources is **less than** the distance between the central maximum and first minimum of any one of them.

Resolving power of grating



It is the capacity of the grating to form separate diffraction maxima of two wavelengths that are close to each other.

The direction of nth principal maximum for wavelength λ is

$(a + b)\ sin\theta_n = n\lambda$ ……..(1)

The equation for minima is $N(a + b)\ sin\theta_n = m\lambda$ where m has all integral values except $0,\ N,\ 2N\ldots..nN$ because for these values of m, the condition for maxima is satisfied.

Thus the first minimum adjacent to nth principal maximum in the direction $\theta_n + d\theta$ is obtained by substituting the value of m as $(nN + 1)$. Thus the first minimum in the direction of $(\theta_n + d\theta)$ is $N(a + b)\sin(\theta_n + d\theta) = (nN + 1)\lambda$ …..(2)

The direction of nth principal maximum for wavelength λ+d λ is

$(a + b)\ sin(\theta_n + d\theta) = n(\lambda + d\lambda)$ …….(3)

Multiplying (3) by N, we have $N(a + b)\ sin(\theta_n + d\theta) = nN(\lambda + d\lambda)$ ….(4)

The two lines appear just resolved if the angle diffraction $(\theta_n + d\theta)$ also correspond to the direction of first secondary minimum due to the first diffraction pattern.

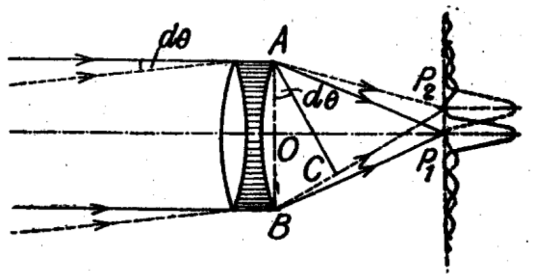
Comparing (2) and (4)

$nN(\lambda + d\lambda) = (nN + 1)\lambda$ or $n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$

$n\ d\lambda = \frac{\lambda}{N}$ or $\frac{\lambda}{d\lambda} = \boldsymbol{nN}$ → Expression for resolving power of grating

Thus the resolving power is (1) directly proportional to the order of the spectrum and (2) the total number of lines on the grating surface.

Resolving power of a telescope



Consider the parallel beam of light striking the lens of the telescope. Path difference between the rays is $BC = BP_2 - AP_2$

From the diagram

$BC = AB\sin d\theta$.

For small angles, $\sin d\theta = d\theta$. Thus $BC = d \times d\theta$ (AB = d).

If $d \times d\theta = \lambda$, P_2 corresponds to first minimum of the first image which is also the position of central maximum of second image.

Thus Raleigh's criterion for resolution is satisfied if $d \times d\theta = \lambda$ or $d\theta = \frac{\lambda}{d}$

According to Airy, this condition in case of a circular aperture is $d\theta = \frac{1.22\lambda}{d}$

$d\theta$ is the minimum resolvable angle between two distinct point objects called limit of resolution. Resolving power is the reciprocal of limit of resolution,

i.e. $RP = \frac{1}{d\theta} = \frac{d}{1.22\lambda}$.

Differences between dispersive power and resolving power of grating

	Dispersive power	Resolving power
1	It provides the angular separation between two spectral lines.	It provides the limit of just resolution of two close objects.
2	$\frac{d\theta}{d\lambda} = \frac{n}{(a + b)\cos\theta}$	$\frac{\lambda}{d\lambda} = nN$
3	If N increases, dispersive power remains same.	If N increases, resolving power also increases.
4	If the grating element $(a + b)$ increases, dispersive power decreases.	If the grating element $(a + b)$ increases, resolving power remains unchanged.

Differences between Prism spectrum and Grating spectrum

	Prism Spectrum	Grating spectrum
1	Due to dispersion – velocities of different colours are different inside the prism.	Due to diffraction – angle of diffraction is different for different wavelengths.
2	Produces only one spectrum	Different orders of spectrum
3	Spectrum is brighter	Spectrum is of less brightness.
4	Deviation is least for red and maximum for violet	Deviation is maximum for red and least for violet
5	Dispersive power $\omega = \frac{dn}{n-1}$ n is refractive index	$\omega = \frac{d\theta}{d\lambda} = \frac{n}{(a + b)\cos\theta}$
6	Spectrum depends on material of	Spectrum independent of material of

	the prism. Resolving power is less	the grating. Resolving power is higher.

Difference between Interference and Diffraction

Interference	Diffraction
1. It is the modification in the intensity of light due to super position of two or more light waves.	1. It is the bending of light around the corners of small obstacles and hence it's spreading into the region of geometrical shadow.
2. It is due to the superposition of finite number of waves from different coherent sources.	2. It is due to the superposition of infinite number of secondary waves from different points of the same wavefront.
3. Interference fringes are of equal width.	3. Diffraction fringes are of unequal width. The width of the central band is maximum and the widths of the less bright bands gradually decrease.
4. Interference pattern consists of alternately bright and dark bands, all the bright bands being of the same brightness.	4. Diffraction pattern consists of a central bright band of maximum brightness, surrounded on either side by alternately dark and less bright bands called secondary maxima.
5. In an interference pattern, a good contrast between dark and bright bands exists. The intensity of dark bands is nearly zero.	5. In a diffraction pattern the contrast between the secondary maxima and minima are comparatively lesser. The intensity of secondary maxima decrease with distance.